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Low-Rank Matrix Recovery using Gabidulin Codes in Characteristic Zero

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Abstract

We present a new approach on low-rank matrix recovery (LRMR) based on Gabidulin Codes. Since most applications of LRMR deal with matrices over infinite fields, we use the recently introduced generalization of Gabidulin codes to fields of characterstic zero. We show that LRMR can be reduced to decoding of Gabidulin codes and discuss which field extensions can be used in the code construction.

Keywords: Gabidulin Codes, Characteristic Zero, Low-Rank Matrix Recovery

1 Introduction

Low-rank matrices occur in many applications, e.g., in signal theory, machine learning and collaborative filtering. Unfortunately, in many cases it is only possible to get incomplete or indirect information of matrices. Since applications usually require complete matrices in order to process data, it is necessary to recover matrices from available data. In general this is not possible.

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However, when matrices are of low-rank, there are efficient algorithms to accomplish this task. So far, *low-rank matrix recovery (LRMR)* is described by a minimization problem which can be solved by convex optimization programs. In this work we show how Gabidulin codes can be used in order to solve the LRMR problem.

2 Low-Rank Matrix Recovery

Low-Rank Matrix Recovery (LRMR) was first defined in [1,2,3] and can be seen as matrix-analogue of compressed sensing [4]. The goal of LRMR is to reconstruct a matrix from incomplete or indirect observations. The problem is stated as follows: We want to recover an unknown matrix $\mathbf{X_0} \in K^{m\times n}$ of lowest possible rank, where in applications K usually is the real or complex field. Therefore, we use observed measurements $\mathbf{y} = \mathcal{A}(\mathbf{X_0})$, which we obtain by applying a linear measurement operator $\mathcal{A} : \mathbb{R}^{m\times n} \to \mathbb{R}^p$ to $\mathbf{X_0}$. We assume that $\mathcal{A}$ can be chosen arbitrarily. Finding a solution to this problem can be specified in terms of the minimization problem

$$\min \operatorname{rank}(\mathbf{X}) \text{ subject to } \mathcal{A}(\mathbf{X}) = \mathcal{A}(\mathbf{X_0}). \quad (1)$$

Often, in literature (1) is also called rank minimization problem. Since this problem is NP-hard, convex relaxations are considered. Most commonly used algorithms are nuclear norm minimization [3] and iterative hard thresholding [5]. An overview of these and other methods, theoretical guarantees and applications is given in [6].

3 Gabidulin Codes in Characteristic Zero

Gabidulin codes over finite fields were introduced in [7,8,9], a comprehensive overview is given in [10]. Since we deal with numbers from infinite alphabets in LRMR, there is a need for Gabidulin codes in characteristic zero, which we introduce according to [11,12,13]. Decoding in rank metric can be described by $\min \operatorname{rank}(\boldsymbol{E}')$ subject to $\mathbf{H}\boldsymbol{E}' = \mathbf{H}\boldsymbol{E}$, which has a similar form as Equation (1). This observation suggests that it might be possible to use a rank metric decoder in order to recover a low-rank matrix and in Section 4 we will show how this can be done.

Let $K \subseteq L$ be fields and L/K be a field extension of degree m. A codeword of a Gabidulin code can either be an $(m \times n)$-matrix over the ground field K or a vector of length n over L. Let $\mathcal{B} = \{\beta_0, \ldots, \beta_{m-1}\}$ be a basis of L over