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About non equivalent completely regular codes with identical intersection array

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Abstract

We obtain several classes of completely regular codes with different parameters, but identical intersection array. Given a prime power q and any two natural numbers a, b, we construct completely transitive codes over different fields with covering radius $\rho = \min\{a, b\}$ and identical intersection array, specifically, we construct one code over $\mathbb{F}_{q^r}$ for each divisor r of a or b. As a corollary, for any prime power q, we show that distance regular bilinear forms graphs can be obtained as coset graphs from several completely regular codes with different parameters.

Keywords: Completely regular codes, coset graphs, distance regular graphs

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1 Introduction

Let $\mathbb{F}_q$ be a finite field of the order q and $\mathbb{F}_q^* = \mathbb{F}_q \setminus \{0\}$. A q-ary linear code C of length n is a k-dimensional subspace of $\mathbb{F}_q^n$. Given any vector $\mathbf{v} \in \mathbb{F}_q^n$, its *distance to the code* C is $d(\mathbf{v}, C) = \min_{\mathbf{x}\in C}\{d(\mathbf{v}, \mathbf{x})\}$, the *minimum distance* of the code is $d = \min_{\mathbf{v}\in C}\{d(\mathbf{v}, C\backslash\{\mathbf{v}\})\}$ and the *covering radius* of the code C is $\rho = \max_{\mathbf{v}\in\mathbb{F}_q^n}\{d(\mathbf{v}, C)\}$. Two vectors $\mathbf{x}$ and $\mathbf{y}$ are *neighbors* if $d(\mathbf{x}, \mathbf{y}) = 1$. We say that C is a $[n, k, d; \rho]_q$-code. Let $D = C + \mathbf{x}$ be a *coset* of C, where $+$ means the component-wise addition in $\mathbb{F}_q$. For a given q-ary code C we define $C(i) = \{\mathbf{x} \in \mathbb{F}_q^n : d(\mathbf{x}, C) = i\}, \quad i = 1, 2, ..., \rho$.

Definition 1.1 [6] A q-ary code C is completely regular, if for all $l \geq 0$ every vector $x \in C(l)$ has the same number c_l of neighbors in $C(l-1)$ and the same number b_l of neighbors in $C(l+1)$. Define $a_l = (q-1)n - b_l - c_l$ and set $c_0 = b_\rho = 0$. Denote by $(b_0, \ldots, b_{\rho-1}; c_1, \ldots, c_\rho)$ the intersection array of C.

Let M be a monomial matrix, i.e. a matrix with exactly one nonzero entry in each row and column. If q is a power of a prime number, then $\mathrm{Aut}(C)$ consist of all monomial $(n \times n)$-matrices M over $\mathbb{F}_q$ such that $\mathbf{c}M \in C$ for all $\mathbf{c} \in C$ and also contains any field automorphism of $\mathbb{F}_q$ which preserves C. The group $\mathrm{Aut}(C)$ acts on the set of cosets of C in the following way: for all $\sigma \in \mathrm{Aut}(C)$ and for every vector $\mathbf{v} \in \mathbb{F}_q^n$ we have $(\mathbf{v} + C)^\sigma = \mathbf{v}^\sigma + C$.

Definition 1.2 [4,10] A linear code C over $\mathbb{F}_q$ with covering radius ρ is completely transitive if $\mathrm{Aut}(C)$ has $\rho + 1$ orbits when acts on the cosets of C.

Since two cosets in the same orbit should have the same weight distribution, it is clear, that any completely transitive code is completely regular.

Completely regular and completely transitive codes are classical subjects in algebraic coding theory, which are closely connected with graph theory, combinatorial designs and algebraic combinatorics. Existence, construction and enumeration of all such codes are open hard problems (see [1,3,5,6] and references there).

In a recent paper [8] we described an explicit construction, based on the Kronecker product of parity check matrices, which provides, for any natural number ρ and for any prime power q, an infinite family of q-ary linear completely regular codes with covering radius ρ. In [9] we presented another class of q-ary linear completely regular codes with the same property, based on lifting of perfect codes. Here, we extend the Kronecker product construction to the case when component codes have different alphabets and connect the resulting completely regular codes with the codes obtained by lifting q-ary