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New parallelisms of $PG(3, 4)$

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Abstract

We construct all 8120217 nonisomorphic parallelisms in $PG(3, 4)$ with automorphisms of order 3 and classify them by the order of their automorphism group and by the type of their spreads.

Keywords: spread, parallelism, regular, authomorphism

1 Introduction

For the basic concepts and notations concerning projective spaces, spreads and parallelisms, refer, for instance, to [4,10] or [16], and for applications to Coding Theory see, for instance, [6] where parallelisms are used in constructions of constant dimension codes that contain lifted MRD codes.

A *spread* in $PG(n, q)$ is a set of lines which partition the point set. A *parallelism* is a partition of the set of lines by spreads. There can be line spreads and parallelisms if n is odd.

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Two parallelisms are *isomorphic* if there exists an automorphism of the projective space which maps each spread of the first parallelism to a spread of the second one.

A subgroup of the automorphism group of the projective space, which maps each spread of the parallelism to a spread of the same parallelism is called *automorphism group* of the parallelism.

A *regulus* of $PG(3, q)$ is a set R of $q+1$ mutually skew lines such that any line intersecting three elements of R intersects all elements of R. A spread S of $PG(3, q)$ is *regular* if for every three distinct elements of S, the unique regulus determined by them is a subset of S. A parallelism is *regular* if all its spreads are regular.

A construction of parallelisms in $PG(n, 2)$ is presented by Zaicev, Zinoviev and Semakov [19] and independently by Baker [1], and in $PG(2^n - 1, q)$ by Beutelspacher [3]. Constructions in $PG(3, q)$ are known due to Denniston [5], Johnson [9], Penttila and Williams [11].

All parallelisms of $PG(3, 2)$ and $PG(3, 3)$ are known [2]. Full classification is currently impossible in the other projective spaces. Computer aided classifications of parallelisms with predefined automorphism groups have been published by Prince [12,13], Stinson and Vanstone [15], Sarmiento [14], Topalova and Zhelezova [17,18], Zhelezova [20].

A parallelism of $PG(3, 4)$ can be obtained by Beutelspacher's general construction [3], and a pair of orthogonal ones by Fuji-Hara's construction for $PG(3, q)$ [7]. The full automorphism group of $PG(3, 4)$ is of order $2^{13}.3^4.5^2.7.17$, so a parallelism might have automorphisms of prime orders $2, 3, 5, 7$ and 17. Order 17 turns up to be impossible. Parallelisms with automorphisms of orders 7 and 5 were classified in [17] and [18]. As a result altogether 32530 parallelisms of $PG(3, 4)$ have been constructed before this work. Only 90 of the 8120217 nonisomorphic parallelisms presented here, possess automorphisms of prime orders greater than 3, the rest are new.

Bamberg announced (`https://symomega.wordpress.com/2012/12/01/`) (2012) that there are no regular parallelisms in $PG(3, 3)$ and $PG(3, 4)$. For $PG(3, 3)$ his result was independently proved by Betten's full classification [2]. No counterexamples are known for $PG(3, 4)$ and we do not find regular ones among the constructed parallelisms either. There are 3 types of spreads in $PG(3, 4)$ - regular, subregular and aregular. We find the types of the spreads of all parallelisms. Ten of them (with a full automorphism group of order 3) have 13 regular spreads, while all previously known parallelisms (with automorphisms of orders 7 or 5) have at most 11 regular spreads.