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One more remark on the adjoint polynomial

Ferenc Bencs

Central European University, Department of Mathematics, H-1051 Budapest, Zrinyi u. 14., Hungary
Alfréd Rényi Institute of Mathematics, H-1053 Budapest Reáltanoda u. 13-15., Hungary

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ABSTRACT

The adjoint polynomial of G is

$$h(G,x)=\sum_{k=1}^{n}(-1)^{n-k}a_k(G)x^k,$$

where $a_k(G)$ denotes the number of ways one can cover all vertices of the graph G by exactly k disjoint cliques of G. In this paper we show the adjoint polynomial of a graph G is a simple transformation of the independence polynomial of another graph $\widehat{G}$. This enables us to use the rich theory of independence polynomials to study the adjoint polynomials. In particular we give new proofs of several theorems of R. Liu and P. Csikvári.

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1. Introduction

Let $a_k(G)$ denote the number of ways one can cover all vertices of the graph G by exactly k disjoint cliques of G. From the definition it is clear that $a_n(G)=1$ and $a_{n-1}(G)=e(G)$, the number of edges, where the number of vertices of G is n. Then the adjoint polynomial of G is

$$h(G,x)=\sum_{k=1}^{n}(-1)^{n-k}a_k(G)x^k.$$

The adjoint polynomial was introduced by R. Liu [8] and it is studied in a series of papers [1,2,12,11,13]. Let us remark that we are using the definition for the adjoint polynomial from [4], but usually it is defined without alternating signs.

E-mail address: ferenc.bencs@gmail.com.

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The adjoint polynomial shows a strong connection with the chromatic polynomial [10]. More precisely the chromatic polynomial of the complement graph $\bar{G}$ of G is

$$ch(\bar{G}, x) = \sum_{k=1}^{n} a_k(G)x(x-1)\dots(x-k+1).$$

The adjoint polynomial shows certain nice analytic properties. For instance it has a real zero whose modulus is the largest among all zeros. H. Zhao showed [11] that the adjoint polynomial always has a real zero, furthermore P. Csikvári proved [4] that the largest real zero has the largest modulus among all zeros. He also showed that the absolute value of the largest real zero is at most $4(\Delta - 1)$, where Δ is the largest degree of the graph G.

Similar results hold for the independence polynomial. Recall that the independence polynomial of a graph H is

$$I(H, x) = \sum_{k\geq 0}(-1)^k i_k(H)x^k,$$

where $i_k(H)$ denotes the number of independent sets of size k in H (note that $i_0(H) = 1$).

Transformations and connections between graph polynomials had already been studied [2,9]. In this paper we will show that there is indeed a connection between the independence and the adjoint polynomials. In particular we will prove the following theorem:

Theorem 1.1. *For any graph G there exists a graph $\widehat{G}$, such that*

$$h(G, x) = x^n I(\widehat{G}, 1/x).$$

This correspondence will enable us to use the rich theory of independence polynomials to study the adjoint polynomials. In particular, we give new proofs of the aforementioned result R. Liu and P. Csikvári. For details see Section 3.

In Section 2 we will give the construction of $\widehat{G}$ and prove Theorem 1.1, moreover, we will show that $\widehat{G}$ can be taken as a spanning subgraph of the line graph of G. Recall that the line graph $L(G)$ for a graph G is a graph on the edge set of G, and there is an edge between two vertices of the line graph if they share a common vertex. This enables us to establish a connection with the matching polynomial of the graph G. For definition of the matching polynomial and applications of this connection see Section 3.

1.1. Notation

Denote the vertex set and edge set of a graph G by $V(G)$ and $E(G)$, respectively. Let $N_G(u)$ denote the set of neighbors of the vertex u and $d(u)$ the degree of the vertex u. Let $N_G[u] = N_G(u) \cup \{u\}$ denote the closed neighborhood of the vertex u. If it is clear from the context, then we will write $N(u)$ and $N[u]$ instead of $N_G(u)$ and $N_G[u]$. Let $G - v$ denote the graph obtained from G by deleting the vertex v. If $S \subseteq V(G)$, then $G[S]$ denotes the induced subgraph of G on the vertex set S, and $G - S$ denotes $G[V(G) - S]$.

2. The construction

In this section we will construct $\widehat{G}$ and prove Theorem 1.1.

Let $\mathcal{K}(G)$ denote the set of clique covers of G, that is

$$\{\{S_1, \dots, S_k\} \subseteq \mathcal{P}(V(G)) \mid \cup_{i=1}^{k} S_i = V(G),\ \text{if } 1 \leq i \neq j \leq n, \text{then} \\ S_i \cap S_j = \emptyset,\ G[S_i] \text{ is a complete graph}\},$$

then one can write the adjoint polynomial of G in the following way:

$$h(G, x) = \sum_{Q\in\mathcal{K}(G)} (-1)^{n-|Q|}x^{|Q|}.$$

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