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Max-cut and extendability of matchings in distance-regular graphs

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ABSTRACT

A connected graph G of even order v is called t -extendable if it contains a perfect matching, $t < v/2$ and any matching of t edges is contained in some perfect matching. The extendability of G is the maximum t such that G is t -extendable. Since its introduction by Plummer in the 1980s, this combinatorial parameter has been studied for many classes of interesting graphs. In 2005, Brouwer and Haemers proved that every distance-regular graph of even order is 1-extendable and in 2014, Cioabă and Li showed that any connected strongly regular graph of even order is 3-extendable except for a small number of exceptions.

In this paper, we extend and generalize these results. We prove that all distance-regular graphs with diameter $D \geq 3$ are 2-extendable and we also obtain several better lower bounds for the extendability of distance-regular graphs of valency $k \geq 3$ that depend on k , λ and μ , where λ is the number of common neighbors of any two adjacent vertices and μ is the number of common neighbors of any two vertices in distance two. In many situations, we show that the extendability of a distance-regular graph with valency k grows linearly in k . We conjecture that the extendability of a distance-regular graph of even order and valency k is at least $\lfloor k/2 \rfloor - 1$ and we prove this fact for bipartite distance-regular graphs.

In course of this investigation, we obtain some new bounds for the max-cut and the independence number of distance-regular graphs in terms of their size and odd girth and we prove that our

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inequalities are incomparable with known eigenvalue bounds for these combinatorial parameters.

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1. Introduction

Our graph theoretic notation is standard (for undefined notions, see [8,26,48]). The adjacency matrix of a graph $G = (V, E)$ has its rows and columns indexed after the vertices of the graph and its (u, v) th entry equals 1 if u and v are adjacent and 0 otherwise. If G is a connected k -regular graph of order v , then k is the largest eigenvalue of the adjacency matrix of G and its multiplicity is 1. In this case, let $k = \lambda_1 > \lambda_2 \geq \dots \geq \lambda_v$ denote the eigenvalues of the adjacency matrix of G . If S and T are vertex disjoint subsets of G , let $e(S, T)$ denote the number of edges with one endpoint in S and the other in T . If S is a subset of vertices of G , let S^c denote its complement. The max-cut of G is defined as $\text{mc}(G) := \max_{S \subseteq V} e(S, S^c)$ and measures how close is G from being a bipartite graph. Given a graph G , determining $\text{mc}(G)$ is a well-known NP-hard problem (see [23, Problem ND16, page 210] or [30]) and designing efficient algorithms to approximate $\text{mc}(G)$ has attracted a lot of attention [1,18–20,27,28,38,44,47].

A set of edges M in a graph G is a matching if no two edges of M share a vertex. A matching M is perfect if every vertex is incident with exactly one edge of M . A connected graph G of even order v is called t -extendable if it contains at least one perfect matching, $t < v/2$ and any matching of size t is contained in some perfect matching. Graphs that are 1-extendable are also called matching-covered (see Lovász and Plummer [36, page 113]). The extendability of a graph G of even order is defined as the maximum $t < v/2$ such that G is t -extendable. This concept was introduced by Plummer [40] in 1980 and was motivated by work of Lovász [34] on canonical decomposition of graphs containing perfect matchings. Later on, Yu [49] expanded the definition of extendability to graphs of odd order. Zhang and Zhang [51] obtained an $O(mn)$ algorithm to compute the extendability of a bipartite graph with n vertices and m edges, but the complexity of determining the extendability of a general graph is unknown at present time (see [41,42,50] for more details on extendability of graphs).

In this paper, we obtain a simple upper bound for the max-cut of certain regular graphs in terms of their odd girth (the shortest length of an odd cycle). In Section 2, we prove that if G is a non-bipartite distance-regular graph with e edges and odd girth g , then $\text{mc}(G) \leq e(1 - \frac{1}{g})$. As a consequence of this result, we show that if G is a non-bipartite distance-regular graph with v vertices, odd girth g and independence number $\alpha(G)$, then $\alpha(G) \leq \frac{v}{2}(1 - \frac{1}{g})$. We show that these bounds are incomparable with some spectral bounds of Mohar and Poljak [38] for the max-cut and of Cvetković (see [8, Theorem 3.5.1] or [26, Lemma 9.6.3]) and Hoffman (see [8, Theorem 3.5.2] or [26, Lemma 9.6.2]) for the independence number.

Holton and Lou [29] showed that strongly regular graphs with certain connectivity properties are 2-extendable and conjectured that all but a few strongly regular graphs are 2-extendable. Lou and Zhu [33] proved this conjecture and showed that every connected strongly regular graph of valency $k \geq 3$ is 2-extendable with the exception of the complete 3-partite graph $K_{2,2,2}$ and the Petersen graph. Cioabă and Li [15] proved that every connected strongly regular graph of valency $k \geq 5$ is 3-extendable with the exception of the complete 4-partite graph $K_{2,2,2,2}$, the complement of the Petersen graph and the Shrikhande graph. Moreover, Cioabă and Li determined the extendability of many families of strongly regular graphs including Latin square graphs, block graphs of Steiner systems, triangular graphs, lattice graphs and all known triangle-free strongly regular graphs. For any such graph of valency k , Cioabă and Li proved that the extendability is at least $\lceil k/2 \rceil - 1$ and conjectured that this fact should be true for any strongly regular graph.

In this paper, we extend and generalize these results and study the extendability of distance-regular graphs with diameter $D \geq 3$. Brouwer and Haemers [7] proved that distance-regular graphs

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