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Unimodular triangulations of simplicial cones by short vectors



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ABSTRACT

We establish a bound for the length of vectors involved in a unimodular triangulation of simplicial cones. The bound is exponential in the square of the logarithm of the multiplicity, and improves previous bounds significantly. The proof is based on a successive reduction of the highest prime divisor of the multiplicity and uses the prime number theorem to control the length of the subdividing vectors.

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1. Introduction

In this paper we discuss the triangulation of simplicial cones C into unimodular subcones by “short” vectors. Length is measured by the basic simplex Δ_C of C that is spanned by the origin and the extreme integral generators of C : we want to find an upper bound for the dilatation factor c for which all subdividing vectors are contained in $c\Delta_C$. Roughly speaking, the larger the multiplicity μ of C (given by the lattice normalized volume of Δ_C), the more subdivision steps are to be expected, and they inevitably increase the length of the subdividing vectors. Therefore μ is the natural parameter on which estimates for c must be based, at least for fixed dimension d .

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A prominent case in which bounds for c come up is the desingularization of toric varieties. The standard argument applied in this situation leads to rather bad bounds. A slight improvement was reached by Bruns and Gubeladze [1, Theorem 4.1] who gave a bound that is better, but still exponential in μ . The main result of this paper is a bound essentially of order $\mu^{\log \mu}$ for fixed dimension d (Corollary 4.6). The next goal would be a bound that is polynomial in μ , but we do not know if such exists.

It seems that the only general technique for triangulating a (simplicial) cone into unimodular subcones is successive stellar subdivision: one chooses an integral vector x in C and replaces C by the collection of subcones that are spanned by x and the facets of C that are visible from C . This simple procedure allows successive refinement of triangulations if simultaneously applied to all cones that contain x .

We start from the basic and rather easy observation that a unimodular triangulation by iterated subdivision can be reached very quickly by short vectors if μ is a power of 2. In order to exploit this observation for arbitrary μ , two crucial new ideas are used: (i) not to diminish the multiplicity μ in every subdivision step (as usually), but to allow it to grow towards a power of 2, and (ii) to control this process by the prime number theorem. While we formulate our results only for simplicial cones, they can easily be generalized (see Remark 4.7.)

The bound in [1, Theorem 4.1] was established in order to prove that multiples cP of lattice polytopes P can be covered by unimodular simplices as soon as c exceeds a threshold that depends only on the dimension d (and not on P or μ). If only unimodular covering is aimed at (for polytopes or cones), one can do much better than for triangulations: the threshold for c has at most the order of d^6 . In particular, it is independent of μ . See Bruns and Gubeladze [2, Theorem 3.23]. A polynomial bound of similar magnitude for the unimodular covering of cones is given in [2, Theorem 3.24]. These polynomial bounds are based on the first part of von Thaden's PhD thesis [5], whereas the results of this paper cover the second part of [5].

The most challenging problem in the area of this paper is to show that the multiples cP of a lattice polytope P have unimodular triangulations for *all* $c \gg 0$. The best known result in arbitrary dimension is the Knudsen–Mumford–Waterman theorem that guarantees the existence of such c . We refer the reader to [2, Chapter 3] and to [3] for an up-to-date survey. The paper [3] contains an explicit upper bound for c .

For unexplained terminology and notation we refer the reader to [2].

2. Auxiliary results

Our first theorem will show that there is a sublinear bound in $\mu(C)$ on the length of the subdividing vectors in a unimodular triangulation if the multiplicity $\mu(C)$ of the cone C is a power of 2. We always assume that a simplicial cone C is generated by its extreme integral generators, i.e., the primitive integral vectors contained in the extreme rays. The multiplicity then is the lattice normalized volume of the simplex Δ_C spanned by them and the origin.

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