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Minimum vertex degree thresholds for tiling complete 3-partite 3-graphs ☆

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A B S T R A C T

Given positive integers $a \leq b \leq c$, let $K_{a,b,c}$ be the complete 3-partite 3-uniform hypergraph with three parts of sizes a, b, c. Let H be a 3-uniform hypergraph on n vertices where n is divisible by $a + b + c$. We asymptotically determine the minimum vertex degree of H that guarantees a perfect $K_{a,b,c}$-tiling, that is, a spanning subgraph of H consisting of vertex-disjoint copies of $K_{a,b,c}$. This partially answers a question of Mycroft, who proved an analogous result with respect to codegree for r-uniform hypergraphs for all $r \geq 3$. Our proof uses a lattice-based absorbing method, the concept of fractional tiling, and a recent result on shadows for 3-graphs.

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1. Introduction

Given $r \geq 2$, an r-uniform hypergraph (in short, *r-graph*) consists of a vertex set V and an edge set $E \subseteq \binom{V}{r}$, that is, every edge is an r-element subset of V. Given an r-graph H with a set S of d vertices, where $1 \leq d \leq r-1$, we define $\deg_H(S)$ to be the number of edges containing S (the subscript H is omitted if it is clear from the context). The *minimum d-degree* $\delta_d(H)$ of H is the minimum of $\deg_H(S)$ over all d-vertex sets S in H. The minimum 1-degree is also referred as the *minimum vertex degree*.

Given two r-graphs F and H, an *F-tiling* (also known as *F-packing*) of H is a collection of vertex-disjoint copies of F in H. An F-tiling is called a *perfect F-tiling* (or an *F-factor*) of H if it covers all the vertices of H. An obvious necessary condition for H to contain an F-factor is $|V(F)| \mid |V(H)|$. Given an integer n that is divisible by $|V(F)|$, we define the *tiling threshold* $t_d(n, F)$ to be the smallest integer t such that every r-graph H of order n with $\delta_d(H) \geq t$ contains an F-factor.

As a natural extension of the matching problem, tiling has been intensively studied in the past two decades (see survey [21]). Much work has been done on graphs ($r = 2$), see e.g., [10,2,19,22]. In particular, Kühn and Osthus [22] determined $t_1(n, F)$, for any graph F, up to an additive constant. Tiling problems become much harder for hypergraphs ($r \geq 3$). For example, despite efforts from many researchers [1,6,13,17,18,23,29,31,32], we still do not know the vertex degree threshold for a perfect matching in r-graphs for arbitrary r.

Other than the matching problem, only a few tiling thresholds are known (see survey [34]) Let K_4^3 denote the complete 3-graph on four vertices, and let $K_4^3 - e$ denote the (unique) 3-graph on four vertices with three edges. Recently Lo and Markström [25] proved that $t_2(n, K_4^3) = (1 + o(1))3n/4$, and Keevash and Mycroft [16] determined the exact value of $t_2(n, K_4^3)$ for sufficiently large n. In [24], Lo and Markström proved that $t_2(n, K_4^3 - e) = (1 + o(1))n/2$. Let $\mathcal{C}_4^3$ be the unique 3-graph on four vertices with two edges (this 3-graph was denoted by $K_4^3 - 2e$ in [5], and by Y in [14]). Kühn and Osthus [20] showed that $t_2(n, \mathcal{C}_4^3) = (1 + o(1))n/4$, and Czygrinow, DeBiasio and Nagle [5] determined $t_2(n, \mathcal{C}_4^3)$ exactly for large n. Recently Mycroft [27] determined $t_{r-1}(n, F)$ asymptotically for many r-partite r-graphs F including all complete r-partite r-graphs and loose cycles.

There are fewer tiling results on vertex degree conditions. Lo and Markström [25] determined $t_1(n, K_3^3(m))$ and $t_1(n, K_4^4(m))$ asymptotically, where $K_r^r(m)$ denotes the complete r-partite r-graph with m vertices in each part. Recently Han and Zhao [15] and independently Czygrinow [4] determined $t_1(n, \mathcal{C}_4^3)$ exactly for sufficiently large n. In this paper we extend these results by determining $t_1(n, K)$ asymptotically for all complete 3-partite 3-graphs K, and thus partially answer a question of Mycroft [26].