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Resonance in orbits of plane partitions and increasing tableaux

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A B S T R A C T

We introduce a new concept of resonance on discrete dynamical systems. This concept formalizes the observation that, in various combinatorially-natural cyclic group actions, orbit cardinalities are all multiples of divisors of a fundamental frequency. Our main result is an equivariant bijection between plane partitions in a box (or order ideals in the product of three chains) under rowmotion and increasing tableaux under K-promotion. Both of these actions were observed to have orbit sizes that were small multiples of divisors of an expected orbit size, and we show this is an instance of resonance, as K-promotion cyclically rotates the set of labels appearing in the increasing tableaux. We extract a number of corollaries from this equivariant bijection, including a strengthening of a theorem of Cameron and Fon-der-Flaass (1995) [9] and several new results on the order of K-promotion. Along the way, we adapt the proof of the conjugacy of promotion and rowmotion from Striker and Williams (2012) [38] to give a generalization in the setting of n-dimensional lattice projections. Finally we discuss known and conjectured examples of resonance relating to alternating sign matrices and fully-packed loop configurations.

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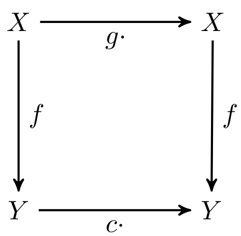
1. Introduction

We introduce the following concept of *resonance*.[1]

Definition 1.1. Suppose $G = \langle g \rangle$ is a cyclic group acting on a set X, $\mathcal{C}_\omega = \langle c \rangle$ a cyclic group of order ω acting nontrivially on a set Y, and $f : X \to Y$ a surjection. We say the triple (X, G, f) exhibits **resonance with frequency** ω if, for all $x \in X$, $c \cdot f(x) = f(g \cdot x)$, that is, the following diagram commutes:



In our examples, Y will be either a set of combinatorial objects drawn in the plane with c acting by rotation or a set of words with c acting by a cyclic shift. Resonance is a *pseudo-periodicity* property of the G-action, in that the resonant frequency ω is generally less than the order of the G-action. Note that (X, G, id_X) satisfies the definition

[1] The mathematically precise definition of resonance given here is new, though the phenomenon has been discussed by various people over the past several years, in particular, at the 2015 "Dynamical Algebraic Combinatorics" workshop at the American Institute of Mathematics where work on this paper began. Thanks to J. Propp for coining the term "resonance" which so nicely encapsulates the idea.