ARTICLE IN PRESS YJCTB:3081

Journal of Combinatorial Theory, Series B ••• (••••) •••–•••

Contents lists available at ScienceDirect

Journal of Combinatorial Theory, Series B

www.elsevier.com/locate/jctb

Asymptotic multipartite version of the Alon–Yuster theorem

Ryan R. Martin [a,1], Jozef Skokan [b,c]

[a] *Department of Mathematics, Iowa State University, Ames, IA 50011, United States*
[b] *Department of Mathematics, London School of Economics, London, WC2A 2AE, UK*
[c] *Department of Mathematics, University of Illinois, 1409 W. Green Street, Urbana, IL 61801, United States*

A R T I C L E I N F O

Article history:
Received 24 July 2013
Available online xxxx

Keywords:
Tiling
Hajnal–Szemerédi
Alon–Yuster
Multipartite
Regularity
Linear programming

A B S T R A C T

In this paper, we prove the asymptotic multipartite version of the Alon–Yuster theorem, which is a generalization of the Hajnal–Szemerédi theorem: If $k \geq 3$ is an integer, H is a k-colorable graph and $\gamma > 0$ is fixed, then, for every sufficiently large n, where $|V(H)|$ divides n, and for every balanced k-partite graph G on kn vertices with each of its corresponding $\binom{k}{2}$ bipartite subgraphs having minimum degree at least $(k-1)n/k + \gamma n$, G has a subgraph consisting of $kn/|V(H)|$ vertex-disjoint copies of H.
The proof uses the Regularity method together with linear programming.

E-mail addresses: rymartin@iastate.edu (R.R. Martin), j.skokan@lse.ac.uk (J. Skokan).

[1] This author's research partially supported by NSF grant DMS-0901008, NSA grant H98230-13-1-0226 and by an Iowa State University Faculty Professional Development grant.

http://dx.doi.org/10.1016/j.jctb.2017.05.004
0095-8956/© 2017 Published by Elsevier Inc.

1. Introduction

1.1. Motivation

One of the celebrated results of extremal graph theory is the theorem of Hajnal and Szemerédi on tiling simple graphs with vertex-disjoint copies of a given complete graph K_k on k vertices. Let G be a simple graph with vertex-set $V(G)$ and edge-set $E(G)$. We denote by $\deg_G(v)$, or simply $\deg(v)$, the degree of a vertex $v \in V(G)$ and we denote by $\delta(G)$ the minimum degree of the graph G. For a graph H such that $|V(H)|$ divides $|V(G)|$, we say that G has a *perfect H-tiling* (also a *perfect H-factor* or *perfect H-packing*) if there is a subgraph of G that consists of $|V(G)|/|V(H)|$ vertex-disjoint copies of H.

The theorem of Hajnal and Szemerédi can be then stated in the following way:

Theorem 1 *(Hajnal, Szemerédi [10]). If G is a graph on n vertices, $k \mid n$, and $\delta(G) \geq (k-1)n/k$, then G has a perfect K_k-tiling.*

The case of $k = 3$ was first proven by Corrádi and Hajnal [5] before the general case. The original proof in [10] was relatively long and intricate. A shorter proof was provided later by Kierstead and Kostochka [16]. Kierstead, Kostochka, Mydlarz and Szemerédi [17] improved this proof and gave a fast algorithm for finding K_k-tilings in n-vertex graphs with minimum degree at least $(k-1)n/k$.

The question of finding a minimum-degree condition for the existence of a perfect H-tiling in the case when H is not a clique and n obeys some divisibility conditions was first considered by Alon and Yuster [1]:

Theorem 2 *(Alon, Yuster [1]). Let H be an h-vertex graph with chromatic number k and let $\gamma > 0$. If n is large enough, $h \mid n$ and G is a graph on n vertices with $\delta(G) \geq (k-1)n/k + \gamma n$, then G has a perfect H-tiling.*

Komlós, Sárközy and Szemerédi [20] removed the γn term from the minimum degree condition and replaced it with a constant that depends only on H.

Kühn and Osthus [23] determined that $(1 - 1/\chi^*(H))\, n + C$ was the necessary minimum degree to guarantee an H-tiling in an n-vertex graph for n sufficiently large, and they also showed that this was best possible up to the additive constant. The constant $C = C(H)$ depends only on H and χ^* is an invariant related to the so-called critical chromatic number of H, which was introduced by Komlós [18].

1.2. Background

In this paper, we consider the multipartite variant of Theorem 2. Before we can state the problem, we need a few definitions.