ARTICLE IN PRESS YJCTB:3083

Journal of Combinatorial Theory, Series B ••• (••••) •••–•••

Contents lists available at ScienceDirect

Journal of Combinatorial Theory, Series B

www.elsevier.com/locate/jctb

Layered separators in minor-closed graph classes with applications ☆

Vida Dujmović [a,1], Pat Morin [b,2], David R. Wood [c,3]

[a] *School of Computer Science and Electrical Engineering, University of Ottawa, Ottawa, Canada*
[b] *School of Computer Science, Carleton University, Ottawa, Canada*
[c] *School of Mathematical Sciences, Monash University, Melbourne, Australia*

ARTICLE INFO

Article history:
Received 14 April 2015
Available online xxxx

Keywords:
Separator
Planar graph
Surface
Minor
Topological minor
Layered separator
Layered treewidth
Nonrepetitive colouring
Queue layout
3-dimensional grid drawing

ABSTRACT

Graph separators are a ubiquitous tool in graph theory and computer science. However, in some applications, their usefulness is limited by the fact that the separator can be as large as $\Omega(\sqrt{n})$ in graphs with n vertices. This is the case for planar graphs, and more generally, for proper minor-closed classes. We study a special type of graph separator, called a *layered separator*, which may have linear size in n, but has bounded size with respect to a different measure, called the *width*. We prove, for example, that planar graphs and graphs of bounded Euler genus admit layered separators of bounded width. More generally, we characterise the minor-closed classes that admit layered separators of bounded width as those that exclude a fixed apex graph as a minor.

We use layered separators to prove $\mathcal{O}(\log n)$ bounds for a number of problems where $\mathcal{O}(\sqrt{n})$ was a long-standing previous best bound. This includes the *nonrepetitive chromatic number* and *queue-number* of graphs with bounded Euler genus. We extend these results with a $\mathcal{O}(\log n)$ bound on the nonrepetitive chromatic number of graphs excluding a fixed topological minor, and a $\log^{\mathcal{O}(1)} n$ bound on the queue-

☆ A short version of this paper and reference [22] was presented at the 54th Annual IEEE Symposium on Foundations of Computer Science (FOCS '13).

E-mail addresses: vida.dujmovic@uottawa.ca (V. Dujmović), morin@scs.carleton.ca (P. Morin), david.wood@monash.edu (D.R. Wood).

[1] Research supported by NSERC and the Ontario Ministry of Research and Innovation.
[2] Research supported by NSERC.
[3] Research supported by the Australian Research Council.

http://dx.doi.org/10.1016/j.jctb.2017.05.006

number of graphs excluding a fixed minor. Only for planar graphs were $\log^{\mathcal{O}(1)} n$ bounds previously known. Our results imply that every n-vertex graph excluding a fixed minor has a *3-dimensional grid drawing* with $n \log^{\mathcal{O}(1)} n$ volume, whereas the previous best bound was $\mathcal{O}(n^{3/2})$.

Contents

1. Introduction

Graph separators are a ubiquitous tool in graph theory and computer science since they are key to many divide-and-conquer and dynamic programming algorithms. Typically, the smaller the separator the better the results obtained. For instance, many problems that are $\mathcal{NP}$-complete for general graphs have polynomial time solutions for classes of graphs that have bounded size separators—that is, graphs of bounded treewidth.

By the classical result of Lipton and Tarjan [53], every n-vertex planar graph has a separator of size $\mathcal{O}(\sqrt{n})$. More generally, the same is true for every proper minor-closed graph class,[4] as proved by Alon et al. [3]. While these results have found widespread use, separators of size $\Theta(\sqrt{n})$, or non-constant separators in general, are not small enough to be useful in some applications.

[4] A graph H is a *topological minor* of a graph G if a subdivision of H is a subgraph of G. A graph H is a *minor* of a graph G if a graph isomorphic to H can be obtained from a subgraph of G by contracting edges. A class $\mathcal{G}$ of graphs is *minor-closed* if $H \in \mathcal{G}$ for every minor H of G for every graph $G \in \mathcal{G}$. A minor-closed class is *proper* if it is not the class of all graphs.

Please cite this article in press as: V. Dujmović et al., Layered separators in minor-closed graph classes with applications, J. Combin. Theory Ser. B (2017), http://dx.doi.org/10.1016/j.jctb.2017.05.006