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Decomposing highly edge-connected graphs into homomorphic copies of a fixed tree

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ABSTRACT

The Tree Decomposition Conjecture by Barát and Thomassen states that for every tree T there exists a natural number $k(T)$ such that the following holds: If G is a $k(T)$-edge-connected simple graph with size divisible by the size of T, then G can be edge-decomposed into subgraphs isomorphic to T. So far this conjecture has only been verified for paths, stars, and a family of bistars. We prove a weaker version of the Tree Decomposition Conjecture, where we require the subgraphs in the decomposition to be isomorphic to graphs that can be obtained from T by vertex-identifications. We call such a subgraph a homomorphic copy of T. This implies the Tree Decomposition Conjecture under the additional constraint that the girth of G is greater than the diameter of T. As an application, we verify the Tree Decomposition Conjecture for all trees of diameter at most 4.

1. Introduction

Let H be a graph. An H-decomposition of a graph G is a partition of its edge-set into subgraphs isomorphic to H. In 2006, Barát and Thomassen [2] made the following conjecture:

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Conjecture 1.1. *For every tree T on m edges, there exists a natural number $k(T)$ such that the following holds:*

If G is a $k(T)$-edge-connected simple graph with size divisible by m, then G has a T-decomposition.

The conjecture trivially holds if T is a single edge. It is easy to see that edge-connectivity 1 suffices for a decomposition into paths of length 2, see for example [11] or [17]. When the conjecture was made, these were the only two cases known to be true. Thomassen [16,17,20] verified the conjecture for paths of length 3 and for paths whose length is a power of 2. Botler, Mota, Oshiro, and Wakabayashi [5,6] proved it for the path of length 5 and also extended the result to paths of any given length. Another proof of the conjecture for paths of any length was found by Bensmail, Harutyunyan, and Thomassé [4].

The results on the weak k-flow conjecture by Thomassen [18] imply that the conjecture holds for all stars. It was further verified for the bistar on 4 edges with degree sequence $(3, 2, 1, 1, 1)$ by Barát and Gerbner [1]. More generally, Thomassen [19] proved the conjecture for all bistars where the degrees of the two non-leaves differ by 1.

The aim of this paper is to prove a weaker version of Conjecture 1.1 with a less restrictive notion of H-decompositions. Let us say that H is a *homomorphic copy* of G if it can be obtained from G by identifying some of its vertices and keeping all edges. Equivalently, H is a homomorphic copy of G if there exists a homomorphism from G to H that is bijective on the edge sets. In particular, a homomorphic copy of G always has the same size as G, and every graph isomorphic to G is also a homomorphic copy of G.

Definition 1.2. Let H and G be graphs. An *H^*-decomposition* of G is a partition of the edge-set of G into homomorphic copies of H.

Clearly every H-decomposition is also an H^*-decomposition. If T is a tree, then a T^*-decomposition is also a T-decomposition provided that the graph we decompose has large girth: If T' is a homomorphic copy of a tree T, but not isomorphic to T, then there exist two vertices u, v in T that have the same image in T'. Since we require the homomorphism to preserve all edges, the path between u and v gets mapped to a closed walk in T'. In particular, there exists a cycle in T' whose length is at most the distance between u and v in T. Thus, if the girth of G is greater than the diameter of T, then every T^*-decomposition of G is also a T-decomposition.

The following weakening of Conjecture 1.1 is the main result of this paper.

Theorem 1.3. *For every tree T on m edges, there exists a natural number $k_h(T)$ such that the following holds:*

If G is a $k_h(T)$-edge-connected graph with size divisible by m, then G has a T^*-decomposition.

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