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Minimum degree conditions for the Hamiltonicity of 3-connected claw-free graphs

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ABSTRACT

Settling a conjecture of Kuipers and Veldman posted in Favaron and Fraisse (2001) [9], Lai et al. (2006) [15] proved that if H is a 3-connected claw-free simple graph of order $n \geq 196$, and if $\delta(H) \geq \frac{n+5}{10}$, then either H is Hamiltonian, or the Ryjáček's closure $cl(H) = L(G)$ where G is the graph obtained from the Petersen graph P by adding $\frac{n-15}{10}$ pendant edges at each vertex of P. Recently, Li (2013) [17] improved this result for 3-connected claw-free graphs H with $\delta(H) \geq \frac{n+34}{12}$ and conjectured that similar result would also hold even if $\delta(H) \geq \frac{n+12}{13}$. In this paper, we show that for any given integer $p > 0$ and real number ϵ, there exist an integer $N = N(p, \epsilon) > 0$ and a family $\mathcal{Q}(p)$, which can be generated by a finite number of graphs with order at most $\max\{12, 3p-5\}$ such that for any 3-connected claw-free graph H of order $n > N$ and with $\delta(H) \geq \frac{n+\epsilon}{p}$, H is Hamiltonian if and only if $H \notin \mathcal{Q}(p)$.

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As applications, we improve both results in Lai et al. (2006) [15] and in Li (2013) [17], and give a counterexample to the conjecture in Li (2013) [17].

1. Introduction

We shall use the notation of Bondy and Murty [1], except when otherwise stated. Graphs considered in this paper are finite and loopless. A graph is called a *multigraph* if it contains multiple edges. A graph without multiple edges is called a *simple graph* or simply a graph. As in [1], $\kappa'(G)$ and $d_G(v)$ (or $d(v)$) denote the edge-connectivity of G and the degree of a vertex v in G, respectively. An edge cut X of a graph G is *essential* if each of the components of $G - X$ contains an edge. A graph G is *essentially k-edge-connected* if G is connected and does not have an essential edge cut of size less than k. An edge $e = uv$ is called a *pendant edge* if either $d(u) = 1$ or $d(v) = 1$. The size of a maximum matching in G is denoted by $\alpha'(G)$. The length of a shortest cycle in G is the *girth* of G. A connected graph G is *Eulerian* if the degree of each vertex in G is even. An Eulerian subgraph Γ in a graph G is called a *spanning Eulerian subgraph* of G if $V(G) = V(\Gamma)$ and is called a *dominating Eulerian subgraph* if $E(G - V(\Gamma)) = \emptyset$. A graph is *supereulerian* if it contains a spanning Eulerian subgraph. The family of supereulerian graphs is denoted by $\mathcal{SL}$. Let $O(G)$ be the set of vertices of odd degree in G. A graph G is *collapsible* if for every even subset $R \subseteq V(G)$, there is a spanning connected subgraph Γ_R of G with $O(\Gamma_R) = R$. When $R = \emptyset$, such Γ_R is a spanning Eulerian subgraph. Examples of collapsible graphs include C_2 (a cycle of length 2) and $K_3 = C_3$. But cycles with length at least 4 (C_i with $i \geq 4$) are not collapsible. We use $\mathcal{CL}$ to denote the family of collapsible graphs. Thus, $\mathcal{CL} \subset \mathcal{SL}$. For a graph G, define $D_i(G) = \{v \in V(G) \mid d(v) = i\}$ and define

$$\bar{\sigma}_2(G) = \min\{d(u) + d(v) \mid \text{for every edge } uv \in E(G)\}. \tag{1}$$

The line graph of a graph G, denoted by $L(G)$, has $E(G)$ as its vertex set, where two vertices in $L(G)$ are adjacent if and only if the corresponding edges in G are adjacent. The following theorem shows a relationship between a graph and its line graph.

Theorem A *(Harary and Nash-Williams [11]). The line graph $H = L(G)$ of a simple graph G with at least three edges is Hamiltonian if and only if G has a dominating Eulerian subgraph.*

A graph H is *claw-free* if H does not contain an induced subgraph isomorphic to $K_{1,3}$. Ryjáček [20] defined the closure $cl(H)$ of a claw-free graph H to be one obtained by recursively adding edges to join two nonadjacent vertices in the neighborhood of any

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