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5-list-coloring planar graphs with distant precolored vertices

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ABSTRACT

We answer positively the question of Albertson asking whether every planar graph can be 5-list-colored even if it contains precolored vertices, as long as they are sufficiently far apart from each other. In order to prove this claim, we also give bounds on the sizes of graphs critical with respect to 5-list coloring. In particular, if G is a planar graph, H is a connected subgraph of G and L is an assignment of lists of colors to the vertices of G such that $|L(v)| \geq 5$ for every $v \in V(G) \setminus V(H)$ and G is not L-colorable, then G contains a subgraph with $O(|H|^2)$ vertices that is not L-colorable.

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1. List colorings of planar graphs

For a graph G, a *list assignment* is a function L that assigns a set of colors to each vertex of G. For $v \in V(G)$, we say that $L(v)$ is the *list* of v. An *L-coloring* of G is a function φ such that $\varphi(v) \in L(v)$ for every $v \in V(G)$ and $\varphi(u) \neq \varphi(v)$ for any pair of adjacent vertices $u, v \in V(G)$. A graph G is *k-choosable* if G is L-colorable for every list assignment L such that $|L(v)| \geq k$ for each $v \in V(G)$.

A well-known result by Thomassen [13] states that every planar graph is 5-choosable. This implies that planar graphs are 5-colorable. Since planar graphs are known to be 4-colorable [2,3], a natural question is whether the result can be strengthened. Voigt [17] gave an example of a non-4-choosable planar graph; hence, the vertices with lists of size smaller than 5 must be restricted in some way. For example, Albertson [1] asked the following question.

Problem 1. Does there exist a constant d such that whenever G is a planar graph with list assignment L that gives lists of size one or five to its vertices and the distance between any pair of vertices with lists of size one is at least d, then G is L-colorable?

For usual colorings, Albertson [1] proved that if S is a set of vertices in a planar graph G that are precolored with colors 1–5 and are at distance at least 4 from each other, then the precoloring of S can be extended to a 5-coloring of G. This solved a problem asked earlier by Thomassen [14].[5] This result does not generalize to 4-colorings even if we have only two precolored vertices (arbitrarily far apart). Examples are given by triangulations of the plane that have precisely two vertices of odd degree. As proved by Ballantine [5] and Fisk [8], the two vertices of odd degree must have the same color in every 4-coloring. Thus, precoloring them with a different color, we cannot extend the precoloring to a 4-coloring of the whole graph.

Recently, there has been significant progress towards the solution of Albertson's problem, see [4] and [7]. Let us remark that when the number of precolored vertices is also bounded by some constant, then the answer is positive by the results of Kawarabayashi and Mohar [9] on 5-list-coloring graphs on surfaces. In this paper, we prove that the answer is positive in general.

Theorem 2. *If G is a planar graph with list assignment L that gives lists of size one or five to its vertices and the distance between any pair of vertices with lists of size one is at least* 20780*, then G is L-colorable.*

In the proof, we need the following result concerning the case that the precolored vertices form a connected subgraph, which is of independent interest.

[5] The problem was posted in the preprint version of [14]; the published version refers to it as being solved already.

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