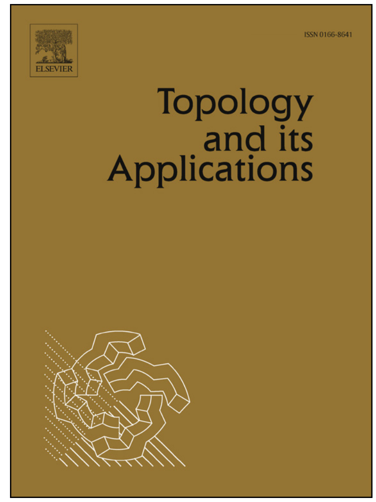

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Abstract

Let $\pi\colon T\times X\to X$, where T is any discrete monoid, be a topological semiflow on a compact Hausdorff space X such that each of its transition maps π_t is a surjection of X. We prove that this semiflow (T,X,π) is equicontinuous if and only if it is uniformly almost periodic if and only if its regionally proximal relation is equal to the diagonal $\varDelta$ of $X\times X$.

Keywords: Equicontinuous semiflow · Uniform almost periodicity · Proximal and regionally proximal· Distal

2010 MSC: 37B20 · 37B05 · 37B35

1. Introduction

Let (G,X) be any transformation group with compact Hausdorff phase space X and with phase group G that consists of continuous self-maps of X such that $e=id_X$ the identity map of X, where e is the neutral element of G. Let $\mathscr{U}_X$ be the symmetric uniformity of X. Then (G,X) is called *equicontinuous* if and only if given any $\varepsilon\in\mathscr{U}_X$ there is $\delta\in\mathscr{U}_X$ such that whenever $x,x'\in X$ with $(x,x')\in\delta$, $(gx,gx')\in\varepsilon$ for all $g\in G$; and (G,X) is called *uniformly almost periodic* if for any $\varepsilon\in\mathscr{U}_X$ there exists a 'relatively dense' subset A of G with $(x,gx)\in\varepsilon$ for all $g\in A$ and $x\in X$.

Then a classical result in topological dynamics, which is due originally to W.H. Gottschalk 1946 [15] and so named "Gottschalk's Theorem" here (cf. [17, Theorem 4.38] and [2, Theorem 2.2]), says that

- (G,X) *is equicontinuous if and only if it is uniformly almost periodic.* (That is why an equicontinuous flow is also called an 'almost periodic flow' in some classical literature.)

However, if G is only a transformation *semigroup* of X, Gottschalk's theorem need not be true obviously. Let us see a simple counterexample as follows.

Example 1.1. Let $X=\{a,b,c\}$ be a discrete phase space consist of three distinct points and let $\xi\colon X\to X$ be defined by $a\mapsto b\mapsto c\mapsto c$. Then the naturally induced transformation semigroup (G,X), where $G=\{\xi^n\,|\,n\in\mathbb{Z}_+\}$, is equicontinuous but not uniformly almost periodic.

We notice here that each transition map $t\in G$ is not a surjection of the phase space X in Example 1.1. So it is natural to ask and consider the following problem.

Question 1.2. *Let (T,X) be a transformation semigroup such that each $t\in T$ is a surjection of X. If (T,X) is equicontinuous, is (T,X) uniformly almost periodic?*

On the other hand, following [11, Definition 6], a pair of points x,y of X is said to be *regionally proximal* for the transformation group (G,X) provided that if U,V are neighborhoods of x,y respectively, and if $\varepsilon\in\mathscr{U}_X$, then there exist $x_1\in U,y_1\in V$, and $g\in G$ such that $(gx_1,gy_1)\in\varepsilon$; equivalently, there exist nets $\{x_i\},\{y_i\}$ in X and $\{g_i\}$ in G

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