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Embedding cones over trees into their symmetric products

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ABSTRACT

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1. Introduction

A continuum is a nonempty, compact, connected metric space. If X is a continuum and n is a positive integer, then C(X) denotes the hyperspace of subcontinua of X and $F_n(X)$ the hyperspace of nonempty subsets of X with at most n points, both endowed with the Hausdorff metric. Given a topological space X, the topological cone of X, denoted by Cone(X), is the quotient space obtained from $X \times [0, 1]$ by shrinking $X \times \{1\}$ to a point, called the vertex of Cone(X), and denoted by ν_X . An element of Cone(X), different from ν_X , will be denoted as an ordered pair (x, t), where $x \in X$ and $t \in [0, 1)$. For convenience, we will consider that the vertex ν_X of Cone(X), is equal to any ordered pair of the form (x, 1), where $x \in X$.

The problem of determining conditions under which Cone(X) is homeomorphic (or topologically equivalent) to C(X) has been studied by many authors. Moreover, some authors have studied the problem of characterizing those continua X for which there exists a finite-dimensional continuum Z such that C(X) is homeomorphic to Cone(Z).

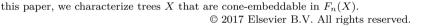
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Given a metric continuum X and a positive integer n, let $F_n(X)$ be the hyperspace

of nonempty sets of X with at most n points and let Cone(X) be the topological

cone of X. We say that a continuum X is cone-embeddable in $F_n(X)$ if there is an

embedding h from Cone(X) into $F_n(X)$ such that $h(x,0) = \{x\}$ for each x in X. In

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Recently, A. Illanes and V. Martínez-de-la-Vega have studied the problem of determining those continua X for which there exists a continuum Z such that $F_n(X)$ is homeomorphic to Cone(Z) ([5]). In particular, they consider the problem when X is a finite graph or a fan (a dendroid with exactly one ramification point) and Z is finite dimensional.

In the rest of the paper, for a continuum X, the symbol $\mathcal{H}(X)$ denotes either C(X) or $F_n(X)$ for some $n \in \mathbb{N}$.

Definition 1. Let X be a continuum. We say that X is **cone-embeddable in** $\mathcal{H}(X)$ if there exists an embedding h: Cone $(X) \to \mathcal{H}(X)$ such that $h(x, 0) = \{x\}$ for each $x \in X$. Such embedding h will be called a **good embedding**.

The third author have studied the case $\mathcal{H}(X) = C(X)$, obtaining general properties of cone-embeddable continua in C(X) ([8]), results on compactifications ([9]) and on arc-smooth continua ([10]). In this paper we work the case $\mathcal{H}(X) = F_n(X)$, when X is a tree and n is a positive integer.

2. Basics definitions and facts

In this section, we present basic concepts and the notation used through the paper. A finite graph is a continuum which can be written as the union of finitely many arcs, called *edges*, any two of which are either disjoint or intersect only in one or both of their end points. The vertices of a finite graph X, are the end points of the edges of X, the set of all vertices of X will be denoted by V(X). A tree is a finite graph containing no simple closed curves. Throughout the paper, G will denote a finite graph and T will denote a tree. Given a positive integer k, a simple k-od is a finite graph which is the union of k arcs emanating from a single point, v, and otherwise disjoint from one another. The point v is called the *core* of the simple k-od.

Given a finite graph $G, p \in G$ and a positive integer k, we say that p is of order k, denoted by $\operatorname{Ord}_G(p) = k$, if p has a closed neighborhood which is homeomorphic to a simple k-od having p as the core. If $\operatorname{Ord}_G(p) = 1$, then p has a neighborhood which is an arc having p as one of its end points and we will call it an *end point* of G. If $\operatorname{Ord}_G(p) = 2$, then p has a neighborhood which is an arc, p is not an end point of it, and we will call it an *ordinary point of* G. A point $p \in G$ is a *ramification point of* G if $\operatorname{Ord}_G(p) \ge 3$. In order to distinguish the vertices from the rest of the points of a finite graph which is not a simple closed curve, we assume that each vertex of G is either an end point of G or a ramification point of G. With this restriction, the two end points of an edge of G may coincide, in whose case such an edge is a simple closed curve. This kind of edges will be called *loops*. We will denote by $\operatorname{Ram}(G)$ the set of all ramification points of G and by E(G) the set of all end points of G.

Given two different vertices $p, q \in G$, we say that p and q are *adjacent* provided that there exists an edge J of G such that p and q are the end points of J. Given a vertex $p \in G$, let

$$N_G(p) = \{q \in V(G) : q \text{ is adjacent to } p\}.$$

Note that for a point $p \in T$, $\operatorname{Ord}_T(p) = |N_T(p)|$, where |A| denotes the cardinality of the set A.

For each pair of distinct vertices $p, q \in T$ we will denote by pq = [p, q] the only arc in T joining p and q, [p,q) will be the set $\{x \in pq : x \neq q\}$. We have in a similar way the natural description of (p,q).

Let G be a finite graph and $p \in \text{Ram}(G)$. We denote by $L_G(p)$ the set of end points adjacent to p in G. We define the ramification order of p in G as $ro_G(p) = |N_G(p) - L_G(p)| = Ord_G(p) - |L_G(p)|$, the number of ramification points adjacent to p in G. For a tree T such that $|\text{Ram}(T)| \ge 2$ we say that a ramification point p of T is exterior if p is an end point of the finite graph $E_p = T - \bigcup\{(p, e] : e \in L_T(p)\}$ obtained by removing all end points adjacent to p together with their corresponding edges; i.e., if $ro_T(p) = 1$. We will denote by Ext(T) the set of exterior points. Observe that the existence of a non exterior ramification point is equivalent to the fact that $|\text{Ram}(T)| \ge 3$. Download English Version:

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