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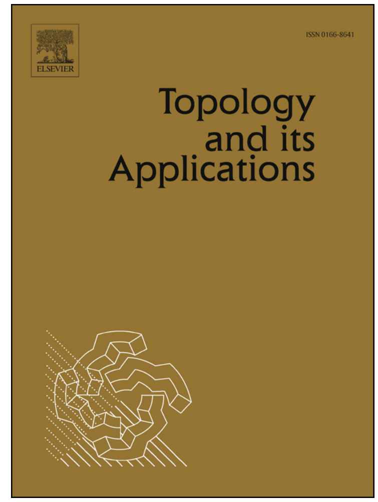

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PROPERTIES AND PRINCIPLES ON PARTIAL METRIC SPACES

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Abstract. In this paper, we topologically study the partial metric space, which may be seen as a new sub-branch of the pure asymmetric topology. We show that many familiar topological properties and principles still hold in certain partial metric spaces, although some results might need some advanced assumptions. Various properties, including separation axioms, countability, connectedness, compactness, completeness and Ekeland's variation principle, are discussed.

1. Introduction and preliminaries

The notion of a partial metric space (PMS) was introduced in 1992 by Steve G. Matthews [1, 2] to model computation over a metric space. The PMS is a generalization of the usual metric space in which the self-distance is no longer necessarily zero. Michael Bukatin, Ralph Kopperman, Steve Matthews, and Homeira Pajoohesh [3] commented that 'a partial metric space combines the metric notion of distance, weight, and poset in a single formalism'. Among hundreds of papers devoted to PMS, most of them were related to the fixed point theorem (see e.g. [6, 7, 8]), a few were related to theoretic computer science (see e.g. [5, 9, 11]). S. Minirani and Sunil Mathew [11] even discussed the nature of fractals in PMS. However, the studies of topological properties of PMS are quite lacking. The presented paper tries to remedy this situation. As a sub-branch of the pure asymmetric topology, PMS possesses many interesting 'asymmetric properties' in practice. In order to understand the structure of PMS better, we shall draw our attention to certain topological properties in this paper.

Definition 1.1. *A partial metric space (see e.g. [1, 2]) is a pair $(X, p : X \times X \to \mathbb{R}^+ \cup \{0\})$ (where $\mathbb{R}^+$ denotes the set of all positive real numbers) such that for all $x, y, z \in X$, we have*

(PM1) $p(x, y) = p(y, x)$;

(PM2) if $0 \le p(x, x) = p(x, y) = p(y, y)$, *then* $x = y$;

(PM3) $p(x, x) \le p(x, y)$;

(PM4) $p(x, z) + p(y, y) \le p(x, y) + p(y, z)$.

We mostly abbreviate 'partial metric space' to PMS, but sometimes, we may even abbreviate 'partial metric spaces' to PMS if it is clear from context.

Apparently, every metric on X is a partial metric on X, and a partial metric p is a metric if and only if $p(x, x) = 0$, $\forall x \in X$. To simplify the notation, let's set $\tilde{p}(x, y) = p(x, y) - p(x, x)$.

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