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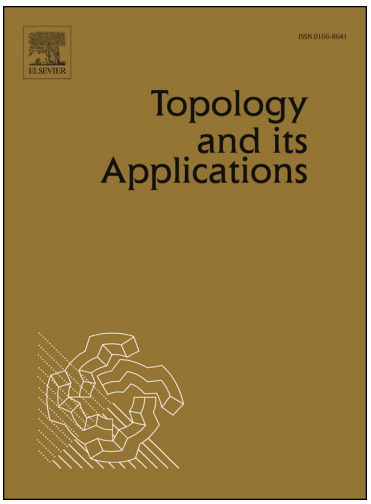

Metrizable-like locally convex topologies on $C(X)$

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Metrizable-like locally convex topologies on $C(X)$

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Abstract

The classic Arens theorem states that the space $C(X)$ of real-valued continuous functions on a Tychonoff space X is metrizable in the compact-open topology τ_k if and only if X is hemicompact. Less demanding but still applicable problem asks whether τ_k has an $\mathbb{N}^{\mathbb{N}}$-decreasing base at zero $(U_\alpha)_{\alpha\in\mathbb{N}^{\mathbb{N}}}$, called in the literature a $\mathfrak{G}$-base. We characterize those spaces X for which $C(X)$ admits a locally convex topology $\mathcal{T}$ between the pointwise topology τ_p and the bounded-open topology τ_b such that $(C(X),\mathcal{T})$ is either metrizable or is an (LM)-space or even has a $\mathfrak{G}$-base.

Keywords: metrizable, (LM)-topology, $\mathfrak{G}$-base, K-analytic, Hewitt realcompactification, functionally bounded set
2010 MSC: 54C35, 46A03

1. Introduction

For a Tychonoff space X we denote by $C_p(X)$, $C_k(X)$ and $C_b(X)$ the space $C(X)$ of all real-valued continuous functions on X endowed with the pointwise topology τ_p, the compact-open topology τ_k and the bounded-open topology τ_b, respectively. By τ_w we mean the weak topology of the locally convex space $C_k(X)$.

The interplay among the topological properties of a Tychonoff space X and the locally convex or topological properties of the space $C(X)$ equipped with a locally convex topology $\mathcal{T}$ has been widely studied, mainly for the cases when $\mathcal{T}$ is τ_p or τ_k. For example, classical Nachbin–Shirota theorems provide necessary and sufficient conditions, in terms of X, for the space $C_k(X)$ to be barrelled or bornological, see [14, Theorems 11.7.5 and 13.6.1]. The corresponding characterizations for $C_p(X)$ are due to Buchwalter and Schmets, see [3].

The question about metrizability of $(C(X),\mathcal{T})$ seems also to be attracting and important. The classic Arens theorem states that $C_k(X)$ is a metrizable (metrizable and complete) locally convex space if and

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