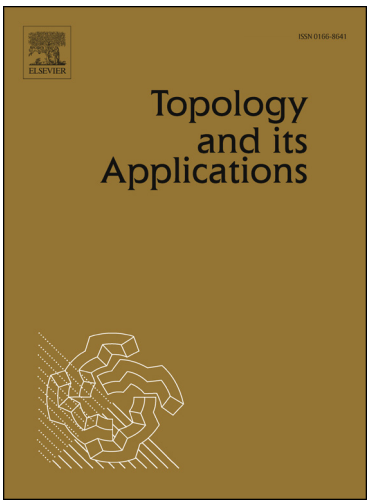

Ribbon-clasp surface-links and normal forms of immersed surface-links

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RIBBON-CLASP SURFACE-LINKS AND NORMAL FORMS OF IMMERSED SURFACE-LINKS

SEIICHI KAMADA AND KENGO KAWAMURA

Dedicated to Professor Taizo Kanenobu, Professor Yasutaka Nakanishi, and Professor Makoto Sakuma in celebration of their 60th birthdays.

Abstract. We introduce the notion of a ribbon-clasp surface-link, which is a generalization of a ribbon surface-link. We generalize the notion of a normal form on embedded surface-links to the case of immersed surface-links and prove that any immersed surface-link can be described in a normal form. It is known that an embedded surface-link is a ribbon surface-link if and only if it can be described in a symmetric normal form. We prove that an immersed surface-link is a ribbon-clasp surface-link if and only if it can be described in a symmetric normal form. We also introduce the notion of a ribbon-clasp normal form, which is a simpler version of a symmetric normal form.

1. Introduction

In this paper an *immersed surface-link* means a closed and oriented surface generically immersed in $\mathbb{R}^4$. When it is embedded, we also call it a *surface-link* or an *embedded surface-link*. An immersed surface-link with one component is called an *immersed surface-knot*. Two immersed surface-links are said to be *equivalent* if they are ambient isotopic.

An embedded surface-link is called a *trivial surface-link* if it is the boundary of a disjoint union of embedded handlebodies in $\mathbb{R}^4$. An embedded surface-link is said to be a *ribbon surface-link* if it is the boundary of immersed handlebodies in $\mathbb{R}^4$ whose multiple point set is a union of ribbon singularities. (The definition of a ribbon singularity is given in Section 2. For an immersion $f : M \to \mathbb{R}^4$ of a bounded 3-manifold M, the *boundary* of the immersed 3-manifold $f(M)$ means the image $f(\partial M)$ of the boundary ∂M of M.)

Definition 1.1. An immersed surface-link is said to be a *ribbon-clasp surface-link* if it is the boundary of immersed handlebodies in $\mathbb{R}^4$ whose

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