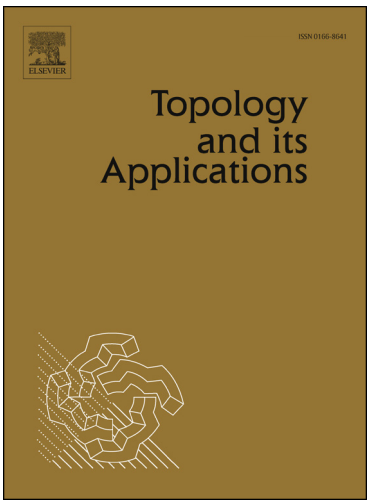

The upper topology and interval topology on quasi-hypercontinuous posets

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The upper topology and interval topology on quasi-hypercontinuous posets[1]

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Abstract

In this paper, we consider a common generalization of both hypercontinuous posets and s_2-quasicontinuous posets, and introduce a new concept of quasi-hypercontinuous posets. The main results are: (1) A poset is a quasi-hypercontinuous poset iff the upper topology is a hypercontinuous lattice iff the $\mathcal{S}^*$-convergence is topological with respect to the upper topology; (2) For a quasi-hypercontinuous poset, the quasi-liminf convergence is topological with respect to the interval topology; (3) The interval topology on a quasi-hypercontinuous poset is Tychonoff; (4) A lattice is quasi-hypercontinuous as a poset iff its normal completion is a quasi-hypercontinuous lattice.

Keywords: Domain theory, Quasi-hypercontinuous poset, Convergence, Upper topology, Interval topology

1. Introduction

The theory of continuous domains, due to its strong background in computer science, general topology and topological algebra, has been extensively studied by people from various areas (see [1, 11]). An important direction in the study of continuous domains is to extend the theory of continuous domains to that of posets as much as possible [5, 17, 18, 20, 23, 25].

As a common generalization of both continuous domains and generalized continuous lattices [12], the concept of a quasicontinuous domain was introduced in [13] by Gierz, Lawson and Stralka. The basic idea is to generalize the way below relation between the points to the case of sets. As a common generalization of both s_2-continuous posets [5] and quasicontinuous domains, we introduced the concept of s_2-quasicontinuous posets by making use of the cut operator instead of joins. The notion of s_2-quasicontinuity admits to generalize most important characterizations of quasicontinuity from dcpos to arbitrary posets and has the advantage that not even the existence of directed joins has to be required [25]. In [12], Gierz and Lawson introduced the concept of a hypercontinuous lattice, which is also among the most successful generalizations of continuous lattices, to characterize a continuous lattice with the Hausdorff interval topology. It was also proved that quasicontinuous domains equipped with the Scott topologies are precisely the spectra of distributive hypercontinuous lattices. In [17], Mao and Xu generalized the concept of hypercontinuity to the setting of general posets.

In this paper, we introduce the concept of quasi-hypercontinuous posets as a common generalization of both hypercontinuous posets and s_2-quasicontinuous posets. It is proved that a poset is a quasi-hypercontinuous poset iff it is an s_2-quasicontinuous poset and the σ_2-topology coincides with the upper topology iff the upper topology is a hypercontinuous lattice iff the $\mathcal{S}^*$-convergence is topological with respect

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