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Induced mappings on the hyperspace of convergent sequences

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A B S T R A C T

The symbol $\mathcal{S}_c(X)$ denotes the hyperspace of all nontrivial convergent sequences in a Hausdorff space X. This hyperspace is endowed with the Vietoris topology. For a given mapping between Hausdorff spaces $f : X \to Y$, define the induced mapping $\mathcal{S}_c(f)$ by $\mathcal{S}_c(f)(A) = f[A]$ (the image of A under f) for every $A \in \mathcal{S}_c(X)$. In the current paper, we study under which conditions the fact that f belongs to a given class of mappings $\mathbb{M}$ implies that $\mathcal{S}_c(f)$ belongs to $\mathbb{M}$, and vice versa.

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1. Introduction

Convergence of sequences is an important tool to determine topological properties in Hausdorff spaces. On the other hand, the study of hyperspaces can provide information about the topological behavior of the original space and vice versa. In connection with both concepts, the hyperspace consisting of all nontrivial convergent sequences $\mathcal{S}_c(X)$, of a metric space X without isolated points, was introduced in [18]. Interesting properties of this hyperspace are presented in [32] and [33] where the study was extended to Hausdorff spaces.

Our main purpose in this paper is to study the following class of continuous functions between hyperspaces. Given a hyperspace $\mathcal{H}(X)$ of a Hausdorff space X and a mapping f from X into a Hausdorff space Y satisfying that $\{f(a) : a \in A\} \in \mathcal{H}(Y)$ for every $A \in \mathcal{H}(X)$, we define the $\mathcal{H}$-*induced mapping* $\mathcal{H}(f) : \mathcal{H}(X) \to \mathcal{H}(Y)$ by $\mathcal{H}(f)(A) = \{f(a) : a \in A\}$.

Now, let $\mathbb{M}$ be a class of mappings between topological spaces. A general problem for a fixed hyperspace $\mathcal{H}(X)$ is to find all possible relationships among the following statements:

(a) $f \in \mathbb{M}$, and
(b) $\mathcal{H}(f) \in \mathbb{M}$.

This problem has been of interest for many authors (see [1–14,20–26,28–31]).

The aim of this paper is to study the interrelations between the statements (a) and (b) when $\mathcal{H}(X) = \mathcal{S}_c(X)$ and $\mathbb{M}$ is each one of the following classes of mappings: open, almost-open, pseudo-open, quotient, closed, surjective, finite-to-1, homeomorphism, monotone, strong light, light, sequence-covering and 1-sequence-covering (see definitions below).

More precisely, we prove that condition (a) is implied by (b) for the following classes of mappings: sequence-covering, surjective, quotient, pseudo-open, almost-open, 1-sequence-covering and finite-to-1; we show the equivalence between (a) and (b) for the classes of mappings: monotone, one-to-one and homeomorphism. We also characterize the mappings for which the induced mapping is open, and finally, we find a condition on the domain of the ground function under which (b) implies (a) for the class of open mappings and for the class of closed mappings. All these results will be presented in Section 4.

Examples in Section 5 show that the converse of most of the results in Section 4 do not hold and also that the hypotheses in some theorems of that section are essential.

2. Preliminaries, definitions and basic results

All topological notions and all set-theoretic notions whose definition is not included here should be understood as in [15] and [27], respectively.

The symbol ω denotes both, the first infinite ordinal and the first infinite cardinal. In particular, we consider all nonnegative integers as ordinals too; thus, $n \in \omega$ implies that $n = \{0, \dots, n-1\}$ and $\omega \setminus n = \{k \in \omega : k \geq n\}$. The successor of ω is the ordinal $\omega + 1 = \omega \cup \{\omega\}$. The set $\omega \setminus \{0\}$ is denoted by $\mathbb{N}$.

If X is a set, $|X|$ will represent the cardinality of X, $[X]^{<\omega}$ is the collection of all finite subsets of X and $[X]^{<n+1}$ is the collection of all subsets of X having at most n elements, whenever $n \in \mathbb{N}$.

For a function f, $\operatorname{ran}(f)$ will denote its range, and given a subset A of the domain of f, the set $\{f(x) : x \in A\}$ will be represented by $f[A]$.

In this paper, *space* means Hausdorff space and *mapping* stands for a continuous function between topological spaces. For a topological space X, the symbol τ_X will denote the collection of all open subsets of X. Also, for a set $A \subseteq X$, we will use $\operatorname{int}_X A$ and $\operatorname{cl}_X A$ to represent its interior in X and its closure in X, respectively. We will get rid of the subscript when we feel there is no risk of confusion regarding our space.