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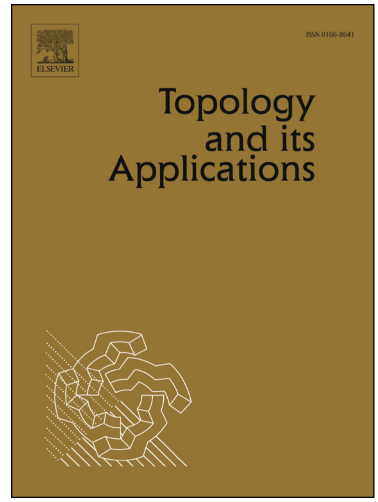

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ON FUNCTIONAL TIGHTNESS OF INFINITE PRODUCTS

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Abstract. A classical theorem of Malykhin says that if $\{X_\alpha : \alpha \leq \kappa\}$ is a family of compact spaces such that $t(X_\alpha) \leq \kappa$, for every $\alpha \leq \kappa$, then $t\left(\prod_{\alpha\leq\kappa} X_\alpha\right) \leq \kappa$, where $t(X)$ is the tightness of a space X. In this paper we prove the following counterpart of Malykhin's theorem for functional tightness: Let $\{X_\alpha : \alpha < \lambda\}$ be a family of compact spaces such that $t_0(X_\alpha) \leq \kappa$ for every $\alpha < \lambda$. If $\lambda \leq 2^\kappa$ or λ is less than the first measurable cardinal, then $t_0\left(\prod_{\alpha<\lambda} X_\alpha\right) \leq \kappa$, where $t_0(X)$ is the functional tightness of a space X. In particular, if there are no measurable cardinals, then the functional tightness is preserved by arbitrarily large products of compacta. Our result answers a question posed by Okunev.

1. Introduction

One of the important cardinal invariants of a topological space X is its tightness $t(X)$ [1]. In this paper we will be interested in the following well-known modification of the tightness arising naturally from the theory of function spaces: Recall that a function $f : X \to Z$ is *κ-continuous* if its restriction to any subset of X of cardinality $\leq \kappa$ is continuous. By $t_0(X)$ we denote the *functional tightness* of X, i.e. the minimal infinite cardinal number κ such that any κ-continuous real-valued function on X is continuous.

A classical theorem of V.I. Malykhin [7] (cf. [1, 2.3.3], [6, 5.9]) asserts that the tightness behaves nicely under infinite Cartesian products of compact spaces. The purpose of the present note is to prove the following counterpart of Malykhin's theorem for functional tightness:

Theorem 1.1. *Let κ be an infinite cardinal and let $\{X_\alpha : \alpha < 2^\kappa\}$ be a family of compact spaces such that $t_0(X_\alpha) \leq \kappa$, for every $\alpha < 2^\kappa$. Then $t_0\left(\prod_{\alpha<2^\kappa} X_\alpha\right) \leq \kappa$.*

This answers a question posed recently by O. Okunev (see [9, Question 3.2]). In fact, we establish a more general result (cf. Corollary 3.7 below), which in particular asserts that if there are no measurable cardinals, then

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[1] *The tightness* $t(X)$ of a space X is the minimal infinite cardinal number κ such that for any $A \subseteq X$ and any $x \in \overline{A}$ there is $C \subseteq A$ with $|C| \leq \kappa$ and $x \in \overline{C}$