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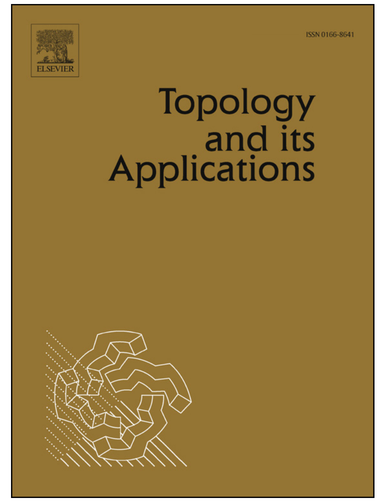

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Universality on frames

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Abstract In the present paper we consider so-called saturated classes of frames of weight less than or equal to a given infinite cardinal τ. Such classes are **saturated** by universal elements. We consider three kinds of saturated classes. The saturated classes of any considered kind have the following basic property: the intersection of not more than τ many saturated classes is also a saturated class and therefore in this intersection there are also universal elements. We prove that the classes $\mathbf{RegFrm}(\tau)$ of regular and $\mathbf{CRegFrm}(\tau)$ of completely regular frames of weight $\leq \tau$ are saturated classes in any of these kinds. This fact implies that in the classes $\mathbf{RegFrm}(\tau, \mu)$ and $\mathbf{CRegFrm}(\tau, \mu)$ of regular and completely regular frames, respectively, of weight $\leq \tau$ and decomposition invariant (see [10]) $\mu \leq \tau$ there are universal elements.

Key words: Lattice, Frame, Universal frame, Saturated class of frames, Decomposition invariant of frames.

2000 Mathematics Subject Classification: 06A06, 06D22, 54A05

1. Introduction

A topological space T is said to be universal in a class $\mathbf{Sp}$ of topological spaces if (a) $T \in \mathbf{Sp}$ and (b) for each $X \in \mathbf{Sp}$ there exists a homeomorphism of X into T. If only the second condition is satisfied, then T is called a *containing space for* $\mathbf{Sp}$. The problem whether there are universal elements actually can be posed for any class of spaces that is determined by a certain topological property. Such problems appeared in topology in its early development, when special classes of separable metrizable spaces were considered. With the consideration of more general classes of spaces, some methods of construction of universal elements appeared. These methods use factorization theorems, separating theorems and product of spaces. In [8] it is described a method of construction of universal and containing spaces in which does not use neither these theorems nor products of spaces. Let $\mathbf{Sp}$ be a class of spaces of weight less than or equal to a fixed infinite cardinal τ. The basic element of the method is the notion of a base $\mathbf{B}$ for a given collection $\mathbf{S}$ of elements of $\mathbf{Sp}$ (which is a fixed base of cardinality $\leq \tau$ in each space of the collection) and a family R of equivalence relations on $\mathbf{S}$. Using these two (fixed) elements, a space denoted by $\mathrm{T}(\mathbf{B}, \mathrm{R})$ is constructed. This space is a containing space for the collection $\mathbf{S}$ of spaces. Suppose that for any collection $\mathbf{S}$ of elements of $\mathbf{Sp}$ some construction containing spaces $\mathrm{T}(\mathbf{B}, \mathrm{R})$ belong to $\mathbf{Sp}$. Then, $\mathrm{T}(\mathbf{B}, \mathrm{R})$ will be a universal element in $\mathbf{Sp}$. In the case where the class $\mathbf{Sp}$ is **saturated** by these kind of elements, this class is called *saturated*.