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Inverse systems and limits in the category of ditopological plain spaces

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ABSTRACT

In this paper, inverse systems (spectrums) and their inverse limits are described and investigated in the some special subcategories of the category whose objects are ditopological texture spaces and morphisms are the point functions satisfying a compatibility condition, besides bicontinuity. According to that, we especially restrict ourselves to the topological subcategory **ifPDitop** consisting of point functions as above and ditopological spaces that have plain texturing which is proper, but still quite extensive subclass of texturings. Followed by, so many major properties of inverse systems and limits in the context of category **ifPDitop** are constructed under the most general conditions. In particular, by considering the full subcategory **ifPDicomp** of **ifPDitop**, whose objects are dicompact plain spaces, we acquired various results about the theory of inverse systems consisting of dicompact plain spaces.

1. Introduction

Our main aim in the present paper, is to give a detailed analysis of the theory of *inverse systems* and their limits under the name *inverse limits* in the context of ditopological plain texture spaces. Accordingly, the foundations of a corresponding inverse system theory in the category whose objects are ditopological

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texture spaces which have plain texturing and morphisms are the bicontinuous point functions satisfying a compatibility condition will be constructed and studied.

This paper consists of five sections and the layout of the paper is as follows:

After presenting some preliminary material required for the paper, in Section 2 mainly, we introduce with the category **ifPDitop** of ditopological plain texture spaces and the bicontinuous point functions satisfying a compatibility condition, called ω*-preserving*.

Following this, Section 3 begins by describing the notion of *inverse system* peculiar to the category **ifPDitop** and also contains another several definitions, examples and results that are important in their own right, and which will also be needed later on. Specifically, this section ends with some relationships between **ifPDitop** and the classical categories **Top** and **Bitop** insofar as the inverse systems are concerned.

Besides, in Section 4 after introducing the notion of *inverse limit* for any inverse system in **ifPDitop** and the other concepts related with the inverse limits inside **ifPDitop**, many useful examples to illustrate the nature of inverse limits are presented. Followed by, the required theorems and results for the rest of paper are mentioned in a categorical setting for plain textures via the notion of joint topology for a ditopology. In this section, a crucial theorem stated in the context of the full subcategory **ifPDicomp** of **ifPDitop**, consisting of dicompact texture spaces is proved, as well as the last theorem based on *the inverse limit map* of the inverse system of mappings.

Following that, Section 5 as the last part of the paper gives a conclusion about the whole of the work.

Indeed the material presented here represents the first stage in the development of the notions *inverse system* and its *inverse limit* for general ditopological texture spaces. Our approach is therefore designed to permit a direct transition from the plain case to the more general cases, for the ditopological texture spaces.

In this paper, generally we have tried to give enough details of the proofs to make it clear where various of the conditions imposed are needed, but at the same time to avoid boring the reader with routine verifications.

According to that, the reader may consult [10] for terms from lattice theory not mentioned here. In addition our standard reference for notions and results from category theory is [1] and if **A** is a category, Ob **A** will denote the class of objects and Mor **A** the class of morphisms of **A**.

Incidentally, an adequate introduction to the theory of ditopological texture spaces and all the motivation for their study, may be obtained from [2–5] and [12–24]. As will be clear from these general standard references, it is showed that ditopological texture spaces provide a unified setting for the study of topology, bitopology and fuzzy topology on Hutton algebras. Especially, there is a close relationship between bitopological and ditopological spaces as shown in [16–18] and [22,23]. Some of the links with Hutton spaces and fuzzy topologies are expressed in a categorical setting in [20]. We will not be interested in the links with fuzzy topology in this paper.

In addition, frequent reference will be made to the author's paper [24] which presents all details related to the subjects *inverse system* and *inverse limit* constructed in the textural context. Otherwise, this paper is largely self-contained. Especially, a significant reference in the general field of inverse system theory as the main topic of this paper is [7].

The remainder of this introduction will be given over to some background material. Hence, we will conclude this section by recalling some preliminary information and results that will enable a casual reader to follow the general ideas presented here.

Inverse systems and inverse limits. The notions of inverse system and (inverse) limit of an inverse system can be defined without a topology on the sets; so these notions are also studied in algebra and analysis. An exhaustive discussion of inverse systems which are in some classical categories such as **Set**, **Top** and **Rng** was presented by [7].

Returning to work at the moment, we will give a detailed analysis of the theory of inverse systems and inverse limits insofar as the category of ditopological plain spaces is concerned, particularly. Also, no