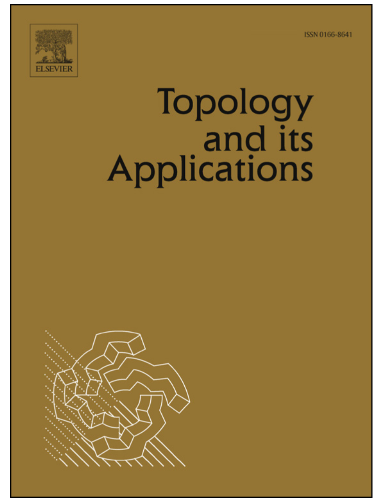

Meir and Keeler were right

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Meir and Keeler were right

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Abstract

It is shown here that the celebrated fixed point theorem of Meir and Keeler is equivalent to formally more general result of Matkowski and Ćirić. In addition, a short proof of an extension of these theorems is given, and a tool theorem with applications is presented. Also some fixed point theorems for commuting (and cyclic) mappings in dislocated metric spaces are proved.

Keywords: Dislocated metric, Partial metric, Fixed point
2000 MSC: 54H25, 47H10

1. Introduction

Meir and Keeler in [6] applied the following condition to a selfmapping f on a metric space:

$$\begin{aligned}&\textit{for each } \alpha > 0, \textit{ there exists an } \epsilon > 0 \textit{ such that}\\ &\alpha \leq p(y,x) < \alpha + \epsilon \textit{ implies } p(fy,fx) < \alpha.\end{aligned} \tag{1}$$

This condition was later extended by Matkowski in [4], Theorem 1.5.1, and by Ćirić [1]. They used two conditions. One of them has the following form:

$$p(fy,fx) < p(y,x), \quad x \neq y.$$

If p is a metric, then the above condition is equivalent to the following one:

$$p(fy,fx) > 0 \textit{ yields } p(fy,fx) < p(y,x), \tag{2}$$

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