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Weight properties in remainders and classes of spaces

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ABSTRACT

A space X belongs to the class $\mathcal{A}$ if for any closed subspace Y of X the network weight $nw(Y)$ is equal to the weight $w(Y)$ of Y. One of the main results of the present article affirms that $bX \setminus X \in \mathcal{A}$ for any compactification bX of the paracompact p-space X. This fact contains a positive answer to one question of A.V. Arhangel'skii and A. Bella. For any Čech-complete space X the above result is not true. Any space with a point-countable k-base is a space from the class $\mathcal{A}$.

1. Introduction

By a space we understand a completely regular topological space. We use the terminology from [16]. Denote by $w(X)$ the weight of a space X. A family $\mathcal{S}$ of subsets of a space X is a network for X if for any point $x \in X$ and any neighborhood U of the point x there exists an element $P \in \mathcal{S}$ such that $x \in P \subseteq U$ (see [1]). The network weight of a space X is the smallest cardinal number of the form $|\mathcal{S}|$, where $\mathcal{S}$ is a network for X and is denoted by $nw(X)$.

The space Y is an extension of X if X is a dense subspace of Y. If Y is a compact space, then Y is a compactification of X. A remainder Z of a space X is the subspace $Z = bX \setminus X$ of a Hausdorff compactification bX of X. One of the major tasks in the theory of compactifications is to find out how the properties of a space X are related to the properties of some or of all remainders of X (see [4,5]).

In this article we consider what kind of remainders a metric space and a paracompact p-space can have. Distinct properties of remainders were studied in [6,9,10,17]. Interesting properties of remainders of metrizable spaces and paracompact p-spaces were described in [4,5,7,8].

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A.V. Arhangel'skii established that $w(X) = nw(X)$ for compact spaces and for p-spaces (see [1,2]). We say that a space X has the Arhangel'skii property (briefly, the Property (α)), if for any closed subspace Y of X we have $nw(Y) = w(Y)$. We denote by $\mathcal{A}$ the class of spaces with the Arhangel'skii property (α).

A family $\mathcal{B}$ of subsets of a space X is called a *k-base* of X if the sets from $\mathcal{B}$ are open in X and for any point $x \in X$ there exists a compact subset $F(x)$ such that $x \in F(x)$ and for each open subset U which contains $F(x)$ we have $x \in V \subseteq U$ for some $V \in \mathcal{B}$. The *k-weight* of a space X is the smallest cardinal number of the form $|\mathcal{B}|$, where $\mathcal{B}$ is a k-base for X and is denoted by $k - w(X)$.

The main results of the present paper are the following theorems:

Theorem 1. *Let X be a space with a point-countable k-base. Then $X \in \mathcal{A}$.*

Theorem 2. *Let X be a dense paracompact p-subspace of a compact space Y. Then $Y \setminus X \in \mathcal{A}$ and, in particular $nw(Z) = w(Z)$ for every closed subspace Z of the remainder $Y \setminus X$.*

In ([8], Corollary 3.6) A.V. Arhangel'skii has proved the following curious statement: If a space P with a countable network is a closed subspace of some remainder $Y \setminus X$ of a paracompact p-space X, then P has a countable base. This fact follows from Theorem 2.

In particular, we obtain the following assertions which contain a positive answer to the Question 2.15 from [9]:

Corollary 1. *Let $X \in \mathcal{A}$ and Y be a dense metrizable subspace of the space X. Then $X \setminus Y \in \mathcal{A}$ and $nw(X \setminus Y) = w(X \setminus Y)$.*

Corollary 2. *If Y is a compactification of a metrizable space X, then $Y \setminus X \in \mathcal{A}$ and $nw(Y \setminus X) = w(Y \setminus X)$.*

Now we mention the following elementary and useful facts:

Proposition 1. *Assume that for each point $x \in X$ there exist an open set $U(x)$ and a subspace $Y(x)$ such that $Y(x) \in \mathcal{A}$ and $x \in U(x) \subseteq Y(x)$. Then $X \in \mathcal{A}$.*

Proof. Let $\mathcal{S}$ be a network of the space X. Then there exists a subset $L \subseteq X$ such that $|L| \leq |\mathcal{S}|$ and $\cup\{U(x) : x \in L\} = X$. Then $w(Y(x)) = nw(Y(x)) \leq |\mathcal{S}|$ and in $U(x)$ there exists an open base $\mathcal{B}(x)$ of cardinality $|\mathcal{B}(x)| \leq |\mathcal{S}|$. Then $\mathcal{B} = \cup\{\mathcal{B}(x) : x \in L\}$ is a base for X and $|\mathcal{B}| \leq |\mathcal{S}|$. Hence $w(X) \leq nw(X)$. The proof is complete.

Corollary 3. *If Y is an open subspace of a space X and $X \in \mathcal{A}$, then $Y \in \mathcal{A}$.*

From the definition it follows:

Corollary 4. *If Y is a closed subspace of a space X and $X \in \mathcal{A}$, then $Y \in \mathcal{A}$.*

2. On spaces with point-countable k-bases

Theorem 3. *For any space X we have $w(X) = k - w(X) + nw(X)$.*

Proof. Obviously, $k - w(X) + nw(X) \leq w(X)$.

Let $k - w(X) + nw(X) \leq \tau$. Assume that τ is an infinite cardinal number and $\mathcal{S}$ is a network of X of the cardinality $\leq \tau$.

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