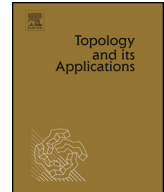




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Nested defining sequences and 1 dimensionality of decomposition elements

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ABSTRACT

It is shown that every cell-like, upper semicontinuous decomposition of an n -manifold, $n \neq 4$, that arises from a nested defining sequence can be replaced by a new cell-like decomposition, with the same quotient space and with elements of dimension at most one.

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This paper addresses the question of whether a cell-like map from an n -manifold to a finite dimensional space can be approximated by another cell-like map having 1-dimensional point preimages. The normal form for decompositions due to Cannon [2] and Quinn [9] (or see [4, Theorem 40.6]) assures this can be done when $n \geq 5$ and X is an n -manifold except for a singular set $S(X)$ of dimension at most $n - 3$. The same conclusion also holds when $S(X)$ is contained in an $(n - 1)$ -manifold. However, the general case remains unresolved.

Here we settle a prominent case in which the map arises as the decomposition map associated with a nested defining sequence in the domain M . The term is explained in the next section. The following is the central result.

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Main Theorem 0.1. *Let M be a compact n -manifold, $n \neq 4$, and G a cell-like upper semicontinuous decomposition of M arising from a nested defining sequence. Then, for each $\epsilon > 0$ the decomposition map $\pi : M \rightarrow X = M/G$ can be ϵ -approximated by a cell-like map $F : M \rightarrow X$ such that each point preimage $F^{-1}(x)$, $x \in X$, has dimension at most 1.*

1. Definitions and basic properties

Let M be an n -manifold. A *nested defining sequence* for M is a sequence $\mathcal{S} = \{\mathcal{M}_1, \mathcal{M}_2, \dots\}$ satisfying the following three conditions:

Disjointness Criterion. For each index i the set \mathcal{M}_i is a locally finite collection $\{A_j\}$ of compact n -manifolds with boundary with pairwise disjoint interiors.

Nested Criterion. For each index $i > 1$, each $A \in \mathcal{M}_i$ has a unique predecessor, $PreA$, in \mathcal{M}_{i-1} that properly contains A .

Boundary Size Criterion. For each index i , each $A \in \mathcal{M}_i$ and each pair of distinct points $x, y \in \partial A$ there is an integer $s > i$ such that no element of \mathcal{M}_s contains both x and y .

These conditions are slightly more general than those of [3] or [4, Section 34], but they are all that are essential.

The decomposition G of M arising from the nested defining sequence $\mathcal{S} = \{\mathcal{M}_1, \mathcal{M}_2, \dots\}$ is the relation given by the rule that $x, y \in M$ belong to the same element of G if there exists a fixed integer k , depending simply on x and y , such that each \mathcal{M}_i has a chain of at most k elements connecting x to y ; explicitly, there is a chain A_1, A_2, \dots, A_k of elements from \mathcal{M}_i such that $x \in A_1$, $y \in A_k$ and $A_j \cap A_{j+1} \neq \emptyset$ for $j = 1, 2, \dots, k-1$. It should be clear that this is an equivalence relation. It will follow from Lemma 1.1 below that equivalent points x, y are actually connected by a chain of length two, but that is not required by the definition.

Since the focus here is on cell-like decompositions, we insist that the nested defining sequence also satisfies the following, which assures that the decomposition arising from the sequence is cell-like; see [3, Theorem 1] or [4, Prop. 34.4].

Null Homotopy Criterion. For $i > 1$ and any $A \in \mathcal{M}_i$ the inclusion $A \rightarrow PreA$ is homotopic to a constant.

Let X be a space and \mathcal{M} a collection of subsets of X . Given $Z \subset X$, define its *star with respect to \mathcal{M}* as

$$St(Z, \mathcal{M}) = Z \cup \{M \in \mathcal{M} \mid M \cap Z \neq \emptyset\}.$$

Set $St^1(Z, \mathcal{M}) = St(Z, \mathcal{M})$ and recursively for each integer $k > 1$ set

$$St^k(Z, \mathcal{M}) = St(St^{k-1}(Z, \mathcal{M}), \mathcal{M}).$$

The following lemma ([3, Addendum to Theorem 1] or [4, Lemma 34.1]) presents important information about the decomposition G .

Lemma 1.1.

- (a) For each $x \in X$, $\pi^{-1}\pi(x) = \cap_i St^2(x, \mathcal{M}_i)$.
- (b) No element $g \in G$ contains more than one point of the set $\partial \mathcal{S}$ defined as $\cup_i \{\partial A \mid A \in \mathcal{M}_i\}$.
- (c) If $x \in g \in G$ and if either $x \in \partial \mathcal{S}$ or $g \cap \partial \mathcal{S} = \emptyset$, then $g = \pi^{-1}\pi(x) = \cap_i St(x, \mathcal{M}_i)$.

Decompositions arising from a nested defining sequence can be rather complex. After all, the concept was introduced to define the Totally Wild Flow on $S^n \times \mathbb{R}$, which involves a cell-like decomposition of S^n , all elements of which are non-cellular, thereby implying that no point of the decomposition space has a manifold

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