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Banach–Stone theorems for spaces of vector bundle continuous sections ☆

Kazuhiro Kawamura

Institute of Mathematics, University of Tsukuba, Tsukuba, Ibaraki 305-8071, Japan

A R T I C L E I N F O

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We prove Banach–Stone type theorems for linear isometries on spaces of continuous sections of Banach space bundles. These extend a previous result for vector-valued continuous function spaces. Two common methods, examining the extreme point sets of the dual spaces and examining peak points, are adopted to prove two similar but slightly different theorems.

1. Introduction and preliminaries

This is a continuation of previous papers [15] and [12] on Banach–Stone type theorems. The classical Banach–Stone theorem states that every surjective linear isometry between the Banach spaces of real/complex valued continuous functions on compact Hausdorff spaces is a unimodular-weighted composition operator (see for example [6]). In [12], some variants of the theorem for Banach space-valued continuous function spaces were discussed. The present paper deals with Banach–Stone type theorems for spaces of continuous sections of Banach space bundles which are extensions of a result [12, Theorem 3.4]. Kuiper's theorem [16] implies that every infinite dimensional Hilbert space bundle is isomorphic to the trivial bundle and hence the extension for such bundles reduces to a previous result. It still seems to be meaningful to study such extension since (i) the results apply to non-trivial bundles, including bundles of Banach spaces for which Kuiper's theorem fails to hold (see [20] for topology of spaces of isometries) and (ii) results are naturally formulated in the context of vector bundle theory (see Theorem 3.1 and Theorem 4.1). Also it lays

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E-mail address: kawamura@math.tsukuba.ac.jp.

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the foundation on the study of Banach–Stone type theorems for “continuous vector fields” over compact Hausdorff spaces. The author recently proved, on the basis of the present paper, Banach–Stone type theorems for isometries on C^1-function spaces over compact Riemannian manifolds [13,14]. A notion of Banach bundles in the context of sheaf theory has been presented in [10], while the present set up is modeled on fiber bundles having unitary groups as their structure groups.

Two methods for proving Banach–Stone theorems are commonly available. The one is to examine the extreme point set of the unit balls of dual spaces making use of the preservation of the sets by adjoint operators of isometries. See for example [2,3,19], and [12]. The other is to examine (weak) peak points making use of the preservation of certain sets of peak points by isometries. See for example [11,5,1,9], and [18]. Also monographs [7] and [8] are invaluable source of information. We adopt both methods to prove two theorems of the same type. The theorems have hypotheses and conclusions which are closely related but slightly different. For the extreme point method we use the scheme as in [15] and [12]. For the peak point method, we adopt the scheme presented in [18]. The author hopes that these shed light on the nature of these methods.

The paper is organized as follows. The rest of this section fixes notation and introduces basics of Banach space bundles. Section 2 identifies the extreme points of dual bundles. Section 3 proves a Banach–Stone type theorem for spaces of Banach space bundle-sections on the basis of results of Section 2. Section 4 adopts the peak function method to prove a theorem of the same type.

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For a Banach space E over a scalar field $\mathbb{F} = \mathbb{R}$ or $\mathbb{C}$, the closed unit ball and the unit sphere are denoted by $B(E)$ and $S(E)$ respectively. For two normed $\mathbb{F}$-linear spaces L_1 and L_2, $\mathcal{L}_{\mathbb{F}}(L_1, L_2)$ denotes the $\mathbb{F}$-linear maps of L_1 to L_2 with the operator norm. An *extreme point* of a convex set C of a linear space L is a point $\xi \in C$ with the property that the equality $\xi = \frac{\eta+\zeta}{2}$ with $\eta, \zeta \in C$ forces $\eta = \zeta = \xi$. The set of all extreme points of C is denoted by $\mathrm{ext}(C)$. In particular the extreme points of the closed unit ball $B(E)$ of a Banach space E is simply denoted by $\mathrm{ext}(E)$. Under this notation, a Banach space E is strictly convex if and only if $\mathrm{ext}(E) = S(E)$ (see [17]).

A *Banach space bundle* over a compact Hausdorff space X is a continuous surjection $\pi : E \to X$ of a topological space E onto a compact Hausdorff space X satisfying the following conditions.

(1) For each $p \in X$, the fiber $E_p = \pi^{-1}(p)$ is a Banach space.
(2) Each point $p \in X$ has an open neighborhood U and a homeomorphism $\lambda : \pi^{-1}(U) \to U \times E_p$ such that
  (2.1) $\mathrm{proj}_U \circ \lambda = \pi|\pi^{-1}(U)$, where $\mathrm{proj}_U : U \times E_p \to U$ denotes the projection onto the factor space U. Hence λ is written as

$$\lambda(e) = (\pi(e), \tau(e)), \quad e \in \pi^{-1}(U)$$

  where $\tau = \mathrm{proj}_{E_p} \circ \lambda$.
  (2.2) For each $q \in U$, the restriction $\tau|E_q : E_q \to E_p$ is an isometric linear isomorphism of the Banach space E_q onto E_p such that $\tau|E_p = \mathrm{id}_{E_p}$.

We sometimes use the terminology “a Banach space bundle E” to indicate a bundle $\pi : E \to X$ when no confusion will occur by the abbreviation. For a point $e \in E$, $\|e\|$ denote the norm of e with respect to $E_{\pi(e)}$. And let $B(E) = \{e \in E \mid \|e\| \leq 1\}$ and $S(E) = \{e \in E \mid \|e\| = 1\}$ denote the *unit disk bundle* and the *unit sphere bundle* of E respectively. A Banach space bundle $\pi : E \to X$ is said to be *strictly convex* (resp. *reflexive*) if each fiber E_p is strictly convex (resp. reflexive).