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Monodromy of torus fiber bundles and decomposability problem

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The notion of a (stably) decomposable fiber bundle is introduced. In low dimensions, for torus fiber bundles over a circle the notion translates into a property of elements of the special linear group of integral matrices. We give a complete characterization of the stably decomposable torus fiber bundle of fiber-dimension less than 4 over the circle.

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1. Introduction

A closed oriented manifold M is said to be decomposable if it is homeomorphic to $M_1 \times M_2$, where M_1 and M_2 are manifolds with $0 < \dim M_i < \dim M$ for $i = 1, 2$. The question of which manifolds are decomposable carries an obvious importance for mathematics. In dimension 1, up to homeomorphism, the only closed and oriented manifold is the unit circle S^1, so, we do not have much ado. The 2-torus $T^2 = S^1 \times S^1$, is decomposable, however, the 2-sphere S^2 is not homeomorphic to a product of two manifolds of dimension 1, hence, it is not decomposable. We call such manifolds indecomposable.

It is easily seen by using elementary properties of Euler characteristic that any closed and oriented surface of genus $g \neq 1$ is indecomposable. However, already in dimension 3 detecting indecomposable manifolds requires some deeper information about the fundamental group of the underlying manifold, and in dimension 4 the problem becomes significantly harder. Moreover, in the astounding world of topological manifolds it may happen that when an indecomposable manifold M is crossed (cartesian product) with

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another (indecomposable) manifold N with $1 \leq \dim N \leq \dim M$ the resulting manifold decomposes into lower dimensional pieces:

$$M \times N \simeq M_1 \times \cdots \times M_r,$$

where $M_1, \ldots, M_r$ are manifolds with $0 < \dim M_i < \dim M$ for $i = 1, \ldots, r$. In this case, N is said to stably decompose M, and M is called stably decomposable. If there is no such N, then M is called stably indecomposable.

In their recent preprint [9], Kwasik and Schultz show that a 3-manifold M is decomposable if and only if it is stably decomposable. Moreover, they show that there exist infinitely many non-homeomorphic indecomposable 4-manifolds M such that $M \times S^k$ is homeomorphic to $S^1 \times S^k \times \mathbb{RP}^3$, where $k = 2, 3, 4$.

The notion of indecomposability applies to different categories. One could consider the decomposability with respect to diffeomorphism, furthermore, one could confine oneself to homogeneous manifolds only. Here, a homogenous manifold means one that admits a transitive action of a Lie group, hence is isomorphic to a coset space. A homogeneous manifold M is called homogeneously decomposable if M is diffeomorphic to a direct product $M_1 \times M_2$ of homogeneous manifolds M_1 and M_2 with $\dim M_i \geq 1$.

Recall that a manifold is called aspherical if its homotopy groups $\pi_k(M)$ vanish for $k \geq 2$. An important class of aspherical manifolds is the class of solvmanifolds. By definition, a *solvmanifold* is a homogeneous space of a connected solvable Lie group. It turns out that two compact solvmanifolds are isomorphic if and only if their fundamental groups are isomorphic, [11]. If M is an aspherical compact manifold with a solvable fundamental group, then it is homeomorphic to a solvmanifold. This follows from the work of Farrell and Jones [3, Corollary B] for $\dim \neq 3$, and from Thurston's Geometrization Conjecture for $\dim = 3$. It follows from these results that if a compact manifold admits a torus fibration over a torus, then it is homeomorphic to a solvmanifold.

In general, the notion of homogeneous decomposability is different from decomposability in differential category. However, it is shown by Gorbatsevich (Proposition 1, [4]) that homogenous decomposability is equivalent to (topological) decomposability as defined in the first paragraph.

The goal of our paper is to define decomposability and carry out a similar analysis in the category of fiber bundles, where fiber product plays the role of the cartesian product. Roughly speaking, we call a bundle over a space B as B-indecomposable if it is indecomposable in the category of fiber bundles over B. This generalizes the previous concepts since the (stable) decomposability of a manifold M is equivalent to (stable) $*$-decomposability of the trivial bundle $M \to M \to *$ over the one point space $*$.

In Section 3, we observe that (stable) indecomposability of the fiber implies (stable) B-indecomposability, hence by the result of Kwasik and Schultz, there exist 4-fiber-dimensional fiber bundles which are B-indecomposable but stably B-decomposable. In Section 4, for low dimensions we interpret the stable decomposability of a torus fiber bundle over a circle in terms of a matrix problem in $\mathrm{SL}(n, \mathbb{Z})$. In Section 5, we show that, for a 3-torus bundle over S^1, the S^1-indecomposability (Theorem 5.2) and the stably S^1-indecomposability (Theorem 5.4) are both determined by the characteristic polynomial of their monodromy matrices. We end our paper by indicating how our results give information about the same problem for torus fibrations over an arbitrary torus as the base space (Remark 5.5).

2. Preliminaries

Standard references for theory of fiber bundles are the textbooks [7] and [13].

For us, a *fiber bundle* is a triplet (F, E, B) of topological spaces along with a surjective map $p : E \to B$ satisfying the following properties:

- F and E are closed, oriented manifolds;
- B is a connected CW-complex;

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