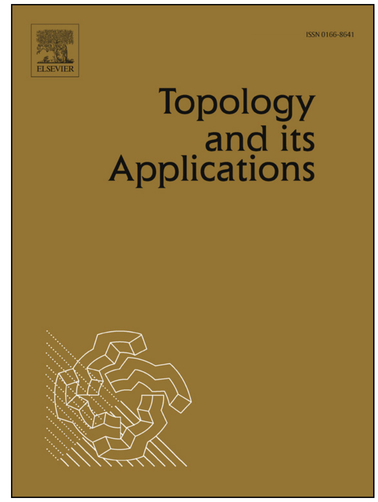
Accepted Manuscript



Non-separable Hilbert manifolds of continuous mappings

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PII: S0166-8641(17)30164-5
DOI: http://dx.doi.org/10.1016/j.topol.2017.03.005
Reference: TOPOL 6095

To appear in: *Topology and its Applications*

Received date: 21 February 2016
Revised date: 4 March 2017
Accepted date: 4 March 2017

Please cite this article in press as: A. Yamashita, Non-separable Hilbert manifolds of continuous mappings, *Topol. Appl.* (2017), http://dx.doi.org/10.1016/j.topol.2017.03.005

NON-SEPARABLE HILBERT MANIFOLDS OF CONTINUOUS MAPPINGS

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Abstract. Let X, Y be separable metrizable spaces, where X is noncompact and Y is equipped with an admissible complete metric d. We show that the space $C(X, Y)$ of continuous maps from X into Y equipped with the uniform topology is locally homeomorphic to the Hilbert space of weight $2^{\aleph_0}$ if (1) (Y, d) is an ANRU, a uniform version of ANR and (2) the diameters of components of Y is bounded away from zero. The same conclusion holds for the subspace $C_B(X, Y)$ of bounded maps if Y is a connected complete Riemannian manifold.

1. Introduction

Most function spaces are infinite-dimensional by nature, and it is natural to ask whether their local structure is similar to that of Hilbert spaces or Banach spaces. From the topological viewpoint, it is known that two Fréchet spaces (i.e., completely metrizable, locally convex real topological vector spaces) are homeomorphic if they have the same weight, due to the characterization of Hilbert spaces by Toruńczyk [18] (cf. [19]). Therefore, any Fréchet space of weight τ is homeomorphic to the *Hilbert space* $\ell^2(\tau)$ of weight τ, in other words, to any Hilbert space with τ orthonormal basis vectors.

Thus we may expect that many function spaces are locally homeomorphic to the Hilbert space $\ell^2(\tau)$ for a suitable τ. Such spaces are known as topological *Hilbert manifolds*, or more precisely, $\ell^2(\tau)$-*manifolds*.

Using the Toruńczyk's characterization mentioned above, many function spaces are known to be Hilbert manifolds. One typical example is the following: the function space $C(X, Y)$ of continuous maps from an infinite compact metric space X into a separable complete metric ANR Y with no isolated points is an ℓ^2-manifold, with respect to the uniform topology (Sakai [15]).[1]

In this paper, we prove corresponding results for $C(X, Y)$ with the uniform topology when X is a *non*compact space, with suitable assumption added to Y. The most significant change from the previous setting is that the function space $C(X, Y)$ becomes non-separable.[2]

When we apply Toruńczyk's characterization to prove a space to be a Hilbert manifold, we have first to prove that the space is an ANR, and this is often difficult. To avoid this problem, we assume that the metric space Y is an *ANRU*, which is a

2010 *Mathematics Subject Classification.* Primary 57N20; Secondary 54C35.

Key words and phrases. function space, non-separable, Hilbert manifold, infinite-dimensional manifold, ANRU, complete Riemannian manifold.

[1]In this case, the uniform topology coincides with many other interesting topologies including the compact-open topology, since X is compact.

[2]In this case, the compact-open topology is coarser than the uniform topology, and $C(X, Y)$ with this topology need not have nice local behavior. This problem is discussed in [17].