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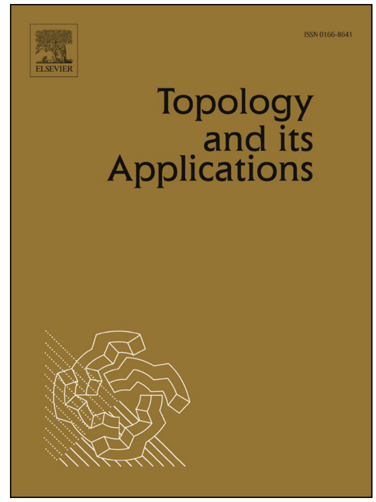

Strict topology on locally compact groups

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Abstract. Let G be a locally compact group, $\mathcal{A}$ a subalgebra of the measure algebra $M(G)$, and $\mathfrak{A}$ a family of Borel subsets of G that is closed under finite unions. In this paper, among other results, we find sufficient conditions on $\mathfrak{A}$, that imply $\mathcal{A}$ is a semi-topological algebra with respect to the strict topology $\beta_{\mathfrak{A}}$. We also find necessary and sufficient conditions on G, that imply $\mathcal{A}$ is a topological algebra with respect to the strict topology $\beta_{\mathfrak{A}}$, where $\mathfrak{A}$ is a family of Borel subsets of G with finite Haar measure.

Introduction and preliminaries

An algebra $\mathcal{A}$ equipped with the topology τ is called a *left* (respectively, *right*)-*topological algebra*, if for each $a \in \mathcal{A}$ the mapping $L_a(x) := ax$ $(x \in \mathcal{A})$ (respectively, $R_a(x) := xa$ $(x \in \mathcal{A})$) is τ-τ-continuous. Also $(\mathcal{A}, \tau)$ is called a *semi-topological algebra* (respectively, *topological algebra*), if the multiplication operation is separately (respectively, jointly) continuous.

Let X be a locally compact space. By Theorem 6.4 of [4], the total variation measure $|\mu|$ of a complex Radon measure μ on X is a finite positive measure. Note that it is not valid for each positive Radon measure, for example the Lebesgue measure on the real line is an infinite positive Radon measure. For a locally compact Hausdorff space X, the set of all complex Radon measures on X with total variation norm is a Banach space, and will be denoted by $M(X)$. Let G be a locally compact group. Then $M(G)$ by the convolution product is a Banach algebra. For $\nu \in M(G)$, let $L_\nu(\mu) := \nu * \mu$ and $R_\nu(\mu) := \mu * \nu$, where $\mu \in M(G)$. A subset $\mathcal{A}$ of $M(G)$ is called *translation invariant* if for each $x \in G$ and $\mu \in \mathcal{A}$, $\delta_x * \mu, \mu * \delta_x \in \mathcal{A}$, where δ_x is the Dirac measure at x.

The organization of this paper is as follows. In section 1, for a locally compact Hausdorff space X, and a $\cup$-closed Borel family $\mathfrak{A}$ (i.e. a family of Borel subsets of X that is closed under finite unions), the topology $\tau_{\mathfrak{A}}$ (i.e. μ_α is $\tau_{\mathfrak{A}}$-convergent to zero, if and only if, for each $A \in \mathfrak{A}$, $|\mu_\alpha|(A) \to 0$) and the *strict topology* $\beta_{\mathfrak{A}}$ (i.e. the locally convex topology generated by seminorms $\mu \to \sup_{n\in\mathbb{N}} \frac{|\mu|(A_n)}{r_n}$, where (A_n) is a sequence in $\mathfrak{A}$, and $0 < r_n \to \infty$) on $M(X)$ are defined. Also the strict continuity of bounded linear maps on $M(X)$ is investigated.

Section 2 is devoted to the strict topology induced by certain class of Borel subsets on a locally compact group, that we call cancellative (i.e. for each $A, B \in \mathfrak{A}$, there exists $C \in \mathfrak{A}$, such that $A^{-1}B, BA^{-1} \subseteq C$). It is shown that the family $\mathfrak{F}$ (the family of all Borel λ-finite subsets of G, where λ is a left Haar measure on G) is cancellative, if and only if, G is compact or discrete. For a subalgebra $\mathcal{A}$

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