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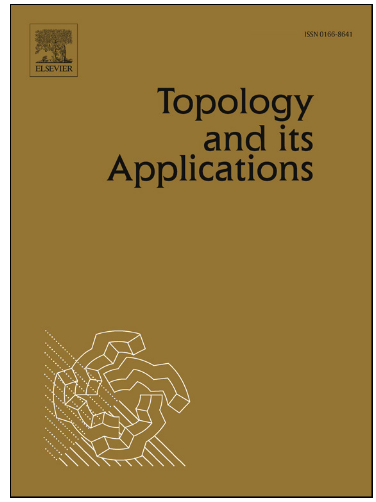
Luis Montejano



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A Variation on the Homological Nerve Theorem

Luis Montejano

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Abstract

An equivalent and useful version of the Homological Nerve Theorem is proved.

1 Introduction

Let X be a polyhedron, $F = \{A_1, \dots A_m\}$ a polyhedral cover of X and let $N = N(F)$ be the nerve of the family F. Denote by $N^{(k)}$ the k-skeleton of the simplicial complex N. In this paper we are going to use reduced homology with coefficients in a field. We say that $A \subset X$ is ρ-acyclic, if $\tilde{H}_*(A) = 0$ for $* \leq \rho$. Furthermore, $\tilde{H}_{-1}(A) = 0$ means A is not empty.

The Homological Nerve Theorem, as stated by Meshulam in [3], claims the following:
Homological Nerve Theorem. Suppose that for every $\sigma \in N^{(k)}$,

- $\bigcap_\sigma$ is $(k- \mid \sigma \mid +1)$-acyclic. Then,

$$\text{Rank } \tilde{H}_{k+1}(N) \leq \text{Rank } \tilde{H}_{k+1}(X),$$

and for every $0 \leq j \leq k$,

$$\tilde{H}_j(N) = \tilde{H}_j(X).$$

The purpose of this paper is to prove the following equivalent, yet utile variation of the Homological Nerve Theorem:

Theorem 1. Suppose that for every $\sigma \in N^{(k)}$,

- $\tilde{H}_{k-|\sigma|+1}(\bigcap_\sigma) = 0$. Then,

1. Rank $\tilde{H}_{k+1}(N) \leq$ Rank $\tilde{H}_{k+1}(X)$,
2. Rank $\tilde{H}_k(X) \leq$ Rank $\tilde{H}_k(N)$.

In many applications of the Homological Nerve Theorem the useful conclusion is that $\tilde{H}_{k+1}(X) = 0$ implies $\tilde{H}_{k+1}(N) = 0$. Thus, Theorem 1 helps to improve these results since the hypotheses needed to achieve this conclusion are much weaker. In Section 3 we give a couple of examples of this fact.