

# Accepted Manuscript

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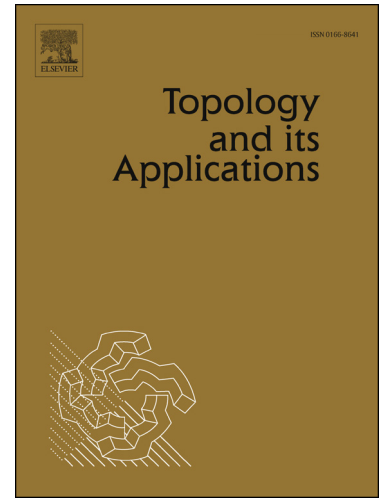
PII: S0166-8641(17)30256-0  
DOI: <http://dx.doi.org/10.1016/j.topol.2017.04.029>  
Reference: TOPOL 6133

To appear in: *Topology and its Applications*

Received date: 12 September 2016  
Revised date: 21 April 2017  
Accepted date: 25 April 2017

Please cite this article in press as: L. Montejano, A Variation on the Homological Nerve Theorem, *Topol. Appl.* (2017), <http://dx.doi.org/10.1016/j.topol.2017.04.029>

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# A Variation on the Homological Nerve Theorem

Luis Montejano

April 27, 2017

## Abstract

An equivalent and useful version of the Homological Nerve Theorem is proved.

## 1 Introduction

Let  $X$  be a polyhedron,  $F = \{A_1, \dots, A_m\}$  a polyhedral cover of  $X$  and let  $N = N(F)$  be the nerve of the family  $F$ . Denote by  $N^{(k)}$  the  $k$ -skeleton of the simplicial complex  $N$ . In this paper we are going to use reduced homology with coefficients in a field. We say that  $A \subset X$  is  $\rho$ -acyclic, if  $\tilde{H}_*(A) = 0$  for  $* \leq \rho$ . Furthermore,  $\tilde{H}_{-1}(A) = 0$  means  $A$  is not empty.

The Homological Nerve Theorem, as stated by Meshulam in [3], claims the following:

**Homological Nerve Theorem.** Suppose that for every  $\sigma \in N^{(k)}$ ,

- $\bigcap_{\sigma}$  is  $(k - |\sigma| + 1)$ -acyclic. Then,

$$\text{Rank } \tilde{H}_{k+1}(N) \leq \text{Rank } \tilde{H}_{k+1}(X),$$

and for every  $0 \leq j \leq k$ ,

$$\tilde{H}_j(N) = \tilde{H}_j(X).$$

The purpose of this paper is to prove the following equivalent, yet utile variation of the Homological Nerve Theorem:

**Theorem 1.** Suppose that for every  $\sigma \in N^{(k)}$ ,

- $\tilde{H}_{k-|\sigma|+1}(\bigcap_{\sigma}) = 0$ . Then,
1.  $\text{Rank } \tilde{H}_{k+1}(N) \leq \text{Rank } \tilde{H}_{k+1}(X)$ ,
  2.  $\text{Rank } \tilde{H}_k(X) \leq \text{Rank } \tilde{H}_k(N)$ .

In many applications of the Homological Nerve Theorem the useful conclusion is that  $\tilde{H}_{k+1}(X) = 0$  implies  $\tilde{H}_{k+1}(N) = 0$ . Thus, Theorem 1 helps to improve these results since the hypotheses needed to achieve this conclusion are much weaker. In Section 3 we give a couple of examples of this fact.

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