

Spherical classes in some finite loop spaces of spheres



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ABSTRACT

We completely determine spherical classes in single, double and triple loop spaces of spheres. We also show that for $l \geq 4$ and $n > 1$, there exists no spherical class in $H_*\Omega^l S^{l+n}$ in dimensions more than $2^l n + 2^{l-1}(l-2) + 2$. This bound may not be optimal, but reduces the computation of spherical classes in $H_*\Omega^l S^{l+n}$ to verifying finite number of cases. We also provide a table for possible spherical classes in $H_*\Omega^l S^{l+n}$ with $3 < l < 9$. Our results, provide positive evidence for the truth of Eccles conjecture.

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1. Introduction

For a pointed space X , let $QX = \text{colim } \Omega^i \Sigma^i X$ be the infinite loop space associated to $\Sigma^\infty X$. Curtis conjecture on spherical classes in H_*QS^0 reads as following.

Conjecture 1.1. ([5, Theorem 7.1]) *For $n > 0$, only Hopf invariant one and Kervaire invariant one elements map nontrivially under the unstable Hurewicz homomorphism*

$${}_{2\pi_n^s} \simeq {}_{2\pi_n} QS^0 \rightarrow H_* QS^0.$$

Throughout the paper, we shall work at the prime 2, the homology will be $\mathbb{Z}/2$ -homology; we write ${}_{2\pi_*^s}$ and ${}_{2\pi_*}$ for the 2-component of π_*^s and π_* respectively. We also write H_* for $H_*(-; \mathbb{Z}/2)$.

A more generalised question, is the determination of the image of the Hurewicz homomorphism $\pi_* \Omega^\infty E \rightarrow H_* \Omega^\infty E$ where E is a suitable spectrum. The aim of this paper is to continue this investigation when $E = \Sigma^\infty S^n$ with $n > 0$ by means of examining the conjecture on finite loops spaces associated to spheres. Let us recall a variant of Curtis conjecture, due to Eccles, which may be stated as follows.

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Conjecture 1.2 (*Eccles conjecture*). Let X be a path connected CW-complex with finitely generated homology. For $n > 0$, suppose $h(f) \neq 0$ where ${}_2\pi_n^s X \simeq {}_2\pi_n QX \rightarrow H_* QX$ is the unstable Hurewicz homomorphism. Then, the stable adjoint of f either is detected by homology or is detected by a primary operation in its mapping cone.

Note that the stable adjoint of f being detected by homology means that $h(f) \in H_* QX$ is stably spherical, i.e. it survives under homology suspension $H_* QX \rightarrow H_* X$ induced by $\Sigma^\infty QX \rightarrow \Sigma^\infty X$ given by the adjoint of the identity $QX \rightarrow X$.

The above conjectures make predictions about the image of spherical classes under the unstable Hurewicz homomorphism $h : {}_2\pi_n^s X \simeq {}_2\pi_n QX \rightarrow H_* QX$. By Freudenthal suspension theorem, for any $f \in \pi_n QX$, depending on the connectivity of X , we may find some nonnegative integer i so that f does pull back to $\pi_n \Omega^i \Sigma^i X$. So, it is natural to ask about the image of the Hurewicz homomorphism $\pi_n \Omega^i \Sigma^i X \rightarrow H_n \Omega^i \Sigma^i X$. Note that for each $i > 0$ there exists an obvious commutative diagram

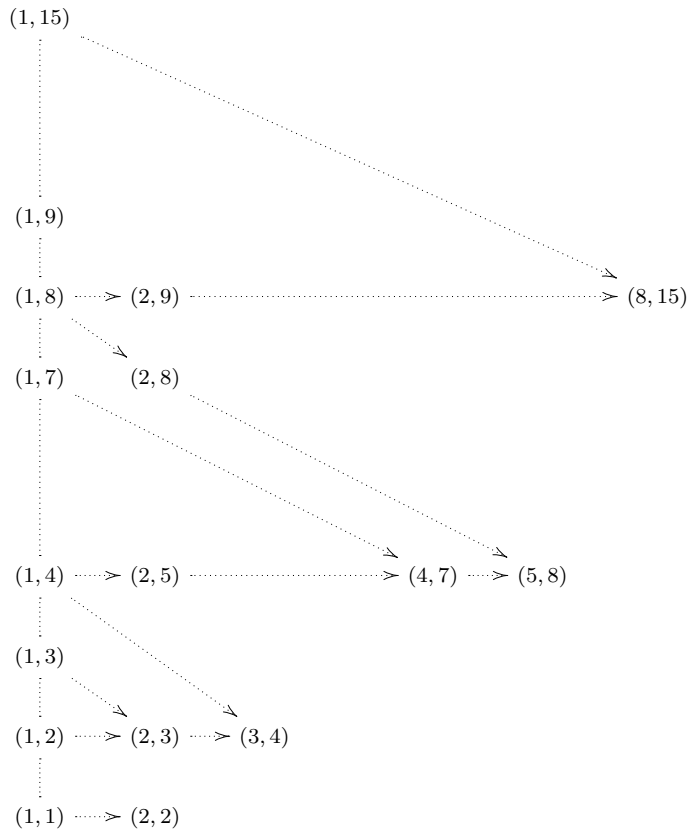
$$\begin{array}{ccc} \pi_n \Omega^i \Sigma^i X & \xrightarrow{h} & H_n \Omega^i \Sigma^i X \\ \downarrow & & \downarrow \\ \pi_n QX & \xrightarrow{h} & H_n QX. \end{array}$$

It is then natural to look for spherical classes in $H_* \Omega^i \Sigma^i X$ for all i in order to give some answer to the above conjectures.

2. Statement of results

Let's recall a related result which was mainly proved by means of geometric tools.

Theorem 2.1. Consider the following lattice where (a, c) corresponds to $H_* \Omega^a S^c$



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