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# Representation space with confluent mappings

José G. Anaya [a], Félix Capulín [a], Enrique Castañeda-Alvarado [a], Włodzimierz J. Charatonik [b,*], Fernando Orozco-Zitli [a]

[a] *Universidad Autónoma del Estado de México, Facultad de Ciencias, Instituto Literario No. 100, Col. Centro, C. P. 50000, Toluca, Estado de México, Mexico*
[b] *Department of Mathematics and Statistics, Missouri University of Science and Technology, Rolla, MO 65409-0020, USA*

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A B S T R A C T

Given a subclass $\mathcal{P}$ of the set $\mathcal{N}$ of all non-degenerate continua we say $X \in \mathrm{Cl}_{\mathcal{F}}(\mathcal{P})$ if for every $\varepsilon > 0$ there are a continuum $Y \in \mathcal{P}$ and a confluent $\varepsilon$-map $f : X \to Y$. This closure operator $\mathrm{Cl}_{\mathcal{F}}$ gives a topology $\tau_{\mathcal{F}}$ on the space $\mathcal{N}$, see [1]. In this article we continue investigation of the topological space $(\mathcal{N}, \tau_{\mathcal{F}})$, we establish interiors and closures of some natural classes of continua, we recall related results and pose several open problems. This gives us a new point of view on topological properties of some classes of continua and on confluent mappings.


## 1. Introduction

Given two topological spaces $X$ and $Y$ and a cover $\mathcal{U}$ of $X$, we say that a mapping $f : X \to Y$ is a $\mathcal{U}$-mapping if there is an open cover $\mathcal{V}$ of $Y$ such that $\{f^{-1}(V) : V \in \mathcal{V}\}$ refines $\mathcal{U}$.

Let $\mathcal{C}$ be a class of topological spaces and let $\alpha$ be a class of mappings between elements of $\mathcal{C}$. We say that $\alpha$ has the composition property if

(1) for every $X \in \mathcal{C}$ the identity map $id_X : X \to X$ is in $\alpha$,
(2) if $f : X \to Y$ and $g : Y \to Z$ are in $\alpha$, then $g \circ f$ is in $\alpha$.

* Corresponding author.
*E-mail addresses:* jgao@uamex.mx (J.G. Anaya), fcapulin@gmail.com (F. Capulín), eca@uaemex.mx (E. Castañeda-Alvarado), wjcharat@mst.edu (W.J. Charatonik), forozcozitli@gmail.com (F. Orozco-Zitli).

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Let $\mathcal{C}$ be a class of topological spaces, let $\mathcal{P}$ be a subset of $\mathcal{C}$, and let $\alpha$ be a class of mappings having the composition property. Given $X \in \mathcal{C}$, we write $X \in \mathrm{Cl}_\alpha(\mathcal{P})$ if for every open cover $\mathcal{U}$ of $X$ there is a space $Y \in \mathcal{P}$ and a $\mathcal{U}$-mapping $f : X \to Y$ that belongs to $\alpha$. The closure operator $\mathrm{Cl}_\alpha$ defines a topology $\tau_\alpha$ in $\mathcal{C}$.

In [1] are proved general properties of the operator $\mathrm{Cl}_\alpha$ and many properties of the topological space $(\mathbb{N}, \tau_\alpha)$, where $\mathbb{N}$ is the space of all non-degenerate metric continua and $\alpha$ is one of the following classes: all mappings, confluent and monotone mappings. Readers specially interested in this topic are referred to [1,5,12]. If $X$ is a metric continuum, $d$ denote a metric in $X$, $d(a, b)$ denote the distance between the points $a$ and $b$ and if $A, B \subset X$, $\mathrm{dist}(A, B)$ denote the distance between the sets $A$ and $B$, defined as the infimum of all distances $d(p, q)$, where $p \in A$ and $q \in B$.

Now in this paper we will give examples of interiors and closures of some classes of continua when $\alpha$ is the family of confluent mappings.

## 2. Definitions, notation and basic results

Let us adopt the following symbols for classes of continua:

| | | |
|---|---|---|
| $\mathbb{AK}$ | — | arc Kelley continua, |
| $\mathbb{D}\mathrm{im}1$ | — | continua of dimension 1, |
| $\mathbb{CF}$ | — | cones over 0-dimensional sets, |
| $\mathbb{D}$ | — | dendroids, |
| $\mathbb{D}_0$ | — | dendrites, |
| $\mathbb{F}$ | — | fans (excluding the arc), |
| $\mathbb{G}$ | — | graphs, |
| $\mathbb{HU}$ | — | hereditarily unicoherent continua, |
| $\mathbb{K}$ | — | Kelley continua, |
| $\mathbb{KT}$ | — | Knaster type continua, including the arc, |
| $\mathbb{LC}$ | — | locally connected continua, |
| $\lambda\mathbb{D}$ | — | $\lambda$-dendroids, |
| $\mathbb{NO}$ | — | $n$-ods, for $n \geq 3$, |
| $\mathbb{SD}$ | — | smooth dendroids, |
| $\mathbb{SF}$ | — | smooth fans, |
| $\mathbb{S}$ | — | solenoids, |
| $\mathbb{TR}$ | — | trees, |
| $\mathbb{TL}$ | — | tree-like continua. |

## 3. Graphs

Let us start with recalling results shown in [1].

**Theorem 3.1.**

1. $\mathrm{Int}_{\mathcal{F}}(\{arc\}) = \{arc\}$,
2. $\mathrm{Cl}_{\mathcal{F}}(\{arc\}) = \mathbb{KT}$,
3. $\mathrm{Int}_{\mathcal{F}}(\{simple\ closed\ curve\}) = \{simple\ closed\ curve\}$,
4. $\mathrm{Cl}_{\mathcal{F}}(\{simple\ closed\ curve\}) = \mathbb{S}$.

The following theorem has been shown in [19, Corollary 3.15, p. 126].