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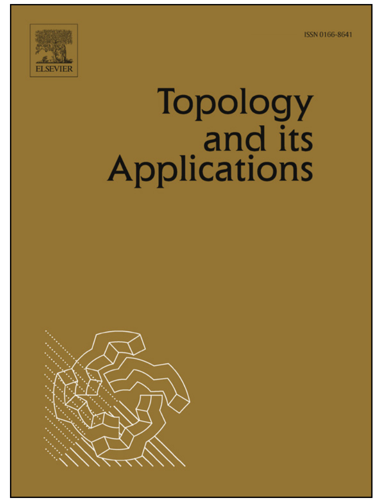

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CONNECTEDNESS AND INVERSE LIMITS WITH SET-VALUED FUNCTIONS ON INTERVALS

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ABSTRACT. In [GK2] it was shown that an inverse limit with upper semicontinuous set-valued functions on intervals is connected if and only if it does not admit a CC-sequence. We introduce a related notion, a component base. Using this new tool we demonstrate how proofs of connectedness theorems can be simplified, and we prove a number of new results on connectedness. In particular, results relating to the size and number of components.

1. INTRODUCTION

It is well known that inverse limits with upper semicontinuous set-valued functions, or generalized inverse limits, can fail to be connected, even when the bonding functions are surjective and have connected graphs, see [I1, Example 2.1] for example. It is also known that a generalized inverse limit over intervals generated by surjective bonding functions with connected graphs must contain a *large* component, one that projects onto each of its graphs (under the appropriate projection) [BK]. Consequently, such inverse limits, at least on intervals, can never be totally disconnected. In [GK1], we characterized a class of generalized inverse limits over intervals that admit clopen basic sets, by C-sequences. In [GK2], we characterised generalised inverse limits over intervals whose bonding functions are surjective and have connected graphs, by CC-sequences (these ideas are discussed below.)

These results leave many questions unanswered. We are interested in the case where the factor spaces are intervals indexed by the nonnegative integers, and we often allow the bonding functions to vary. We require that the bonding functions $f_i : I_i \to 2^{I_{i-1}}$ (for $i > 0$) be upper semicontinuous, surjective, and have connected graphs. We introduce the notion of a component base, and show that the existence of a component base is equivalent to the existence of a CC-sequence. We give new results on connectedness of inverse limits and of Mahavier products, and revisit known results that can be simplified by using component bases. In particular:

- We answer the following question posed by Tom Ingram:
  > [I1, Problem 6.6] What can be said about compacta that are inverse limits with a single upper semicontinuous function whose graph is the union of two maps without a coincidence point?
- We show that:
  - any inverse limit has exactly one large component;
  - an inverse limit can have $\mathfrak{c}$ many small components; and

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