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More absorbers in hyperspaces

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ABSTRACT

The family of all subcontinua that separate a compact connected n-manifold X (with or without boundary), $n \geq 3$, is an F_σ-absorber in the hyperspace $C(X)$ of nonempty subcontinua of X. If $D_2(F_\sigma)$ is the small Borel class of spaces which are differences of two σ-compact sets, then the family of all $(n-1)$-dimensional continua that separate X is a $D_2(F_\sigma)$-absorber in $C(X)$. The families of nondegenerate colocally connected or aposyndetic continua in I^n and of at least two-dimensional or decomposable Kelley continua are $F_{\sigma\delta}$-absorbers in the hyperspace $C(I^n)$ for $n \geq 3$. The hyperspaces of all weakly infinite-dimensional continua and of C-continua of dimensions at least 2 in a compact connected Hilbert cube manifold X are Π_1^1-absorbers in $C(X)$. The family of all hereditarily infinite-dimensional compacta in the Hilbert cube I^ω is Π_1^1-complete in 2^{I^ω}.

1. Introduction

The theory of absorbing sets was well developed in the eighties and nineties of the last century (see [4] and [33]). Since any two absorbers in a Hilbert cube $\mathcal{Q}$ of a given Borel or projective class are homeomorphic via arbitrarily small ambient homeomorphisms of $\mathcal{Q}$, it provides a powerful technique of characterizing some subspaces of the hyperspaces $\mathcal{Q} = 2^X$ of all closed nonempty subsets of a nondegenerate Peano continuum X

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or $\mathcal{Q} = C(X)$ of all subcontinua of X (if X contains no free arcs). Nevertheless, the list of natural examples of subspaces of $C(X)$ studied in continuum theory which are absorbers is not too long.

Let $I = [0,1]$ be the closed unit interval with the Euclidean metric. The following subspaces of respective hyperspaces are absorbers of the Borel class F_σ (otherwise known as cap-sets), so they are homeomorphic to the pseudo-boundary $B(I^\omega) = \{(x_i) \in I^\omega : \exists\, i\,(x_i \in \{0,1\})\}$, a standard F_σ-absorber in the Hilbert cube I^ω:

Examples 1.1.

1. The subspaces $\mathcal{D}_n(X)$ of 2^X consisting of all compacta of covering dimensions $\geq n \geq 1$ and $\mathcal{D}_{n+1}(X) \cap C(X)$ of $C(X)$, where X is a locally connected continuum each of whose open non-empty subset has dimension $\geq n$ [7].
   Initially, the absorbers were obtained for $X = I^\omega$ or X being the infinite product of nondegenerate locally connected continua and it was observed that the covering dimension can be replaced by more general integer-valued dimension functions (so called true dimensions) including cohomological dimensions [14,16,18,19,21] and even by real-valued dimension functions such as the Hausdorff dimension [3,31].
2. The family of all decomposable continua in I^n, $n \geq 3$ [38].
3. The family of all compact subsets (subcontinua) with nonempty interiors in a locally connected nondegenerate continuum (containing no free arcs) [12].
4. The family of all compact subsets that block all subcontinua of a locally connected nondegenerate continuum which is not separated by any finite subset [22].
5. The subspace $\mathcal{F}(X) \subset 2^X$ of all finite subsets of X for $X = I^\omega$ [11]. In general, $\mathcal{F}(X)$ is not an F_σ-absorber in 2^X, as it is homeomorphic to the strongly countable-dimensional linear subspace l_2^f of l_2 spanned by the canonical orthonormal basis of l_2 if X is non-degenerate, connected, locally path-connected, strongly countable-dimensional and σ-compact [10]. The latter result was extended to non-separable spaces X, see [24,34,40].

Examples 1.2. Known $F_{\sigma\delta}$-absorbers include two standard ones $(B(I^\omega))^\omega$ (in $(I^\omega)^\omega$) and $\widehat{c_0} = \{(x_i) \in I^\omega : \lim_i x_i = 0\}$ (in I^ω) [14] and

1. The subspace of 2^X $(C(X))$ of all infinite-dimensional (with respect to a true dimension) compacta (continua) for X being the countable infinite product of nondegenerate locally connected continua [14,16,18,19,21].
2. The subspace of $C(I^n)$ of all locally connected subcontinua of I^n, $n \geq 3$ [20].
3. The subspaces of $C(I^2)$ of all arcs [6] and of all absolute retracts [8].

Examples 1.3. If $D_2(F_\sigma)$ is the class of all subsets of the Hilbert cube that are differences of two F_σ-sets, then a standard $D_2(F_\sigma)$-absorber is the subset $B(I^\omega) \times s$ of $I^\omega \times I^\omega$ (where $s = I^\omega \setminus B(I^\omega)$); its incarnation in 2^{I^ω} is the family $\mathcal{D}_n(I^\omega) \setminus \mathcal{D}_{n+1}(I^\omega)$, $n \geq 1$ [14].

Example 1.4. The subspace of $2^{\mathbb{R}^n}$, $n \geq 3$, of all compact ANR's in $\mathbb{R}^n$ is a $G_{\delta\sigma\delta}$-absorber [15] (recall that if $n \geq 2$ then the hyperspace $2^{\mathbb{R}^n}$ is homeomorphic to $I^\omega \setminus \{\text{point}\}$ [9]).

Let Π_1^1 and Σ_1^1 denote the classes of coanalytic and analytic sets, respectively. Concerning Π_1^1-absorbers (coanalytic absorbers), the Hurewicz set $\mathcal{H}$ of all countable closed subsets of I can be treated as a standard Π_1^1-absorber in the hyperspace 2^I [5]. Other known examples of coanalytic absorbers we wish to recall here are: