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Variations of selective separability and tightness in function spaces with set-open topologies ☆

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A B S T R A C T

We study tightness properties and selective versions of separability in bitopological function spaces endowed with set-open topologies.

1. Introduction

In this paper we are mainly concerned with selective versions of separability in bitopological function spaces endowed with two homogenous set-open topologies.

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Variations of separability, stronger forms, weaker forms, functional separability and similar properties have been intensively studied by many mathematicians in the last two decades. The selective versions of separability have recently gained a particular attention and many interesting results were obtained.

Although the definition of selection principles was given by Scheepers in 1996, the theory is actually based on the papers by Menger, Hurewicz, Rothberger and Sierpiński in 1920–1930, see [11,17,25].

Many topological properties can be defined or characterized in terms of the following two classical selection principles given in a general form in [28] as follows:

Let $\mathcal{A}$ and $\mathcal{B}$ be sets consisting of families of subsets of an infinite set X. Then:

$S_1(\mathcal{A},\mathcal{B})$: for each sequence $(A_n : n \in \mathbb{N})$ of elements of $\mathcal{A}$ there is a sequence $(b_n : n \in \mathbb{N})$ such that for each n, $b_n \in A_n$, and $\{b_n : n \in \mathbb{N}\}$ is an element of $\mathcal{B}$.

$S_{fin}(\mathcal{A},\mathcal{B})$: for each sequence $(A_n : n \in \mathbb{N})$ of elements of $\mathcal{A}$ there is a sequence $(B_n : n \in \mathbb{N})$ of finite sets such that for each n, $B_n \subseteq A_n$, and $\bigcup_{n\in\mathbb{N}} B_n \in \mathcal{B}$.

The selection principles denoted by $S_{fin}(\mathcal{O},\mathcal{O})$ and $S_1(\mathcal{O},\mathcal{O})$ are called the Menger and Rothberger property, where $\mathcal{O}$ is the family of open covers of a topological space.

For the topological space X, let $\mathcal{D}$ denote the family of dense subspaces of X. The selection principles $S_{fin}(\mathcal{D},\mathcal{D})$ and $S_1(\mathcal{D},\mathcal{D})$ were introduced by Scheepers in [29] and recently gained a great attention, see [3–5, 10].

In [3] the selection properties $S_{fin}(\mathcal{D},\mathcal{D})$, $S_1(\mathcal{D},\mathcal{D})$ and $S_1(\mathcal{D},\mathcal{D}^{gp})$ are called M-separability (also called selective separability), R-separability and GN-separability, respectively, while a modified property $S_{fin}(\mathcal{D},\mathcal{D}^{gp})$ is called H-separability where "M-", "R-" and "H-" represent well known Menger, Rothberger and Hurewicz properties.

It should be noted that very recently Tsaban and his co-authors in [6] studied all properties $S(\mathcal{A},\mathcal{B})$ for $S \in \{S_1, S_{fin}\}$ and $\mathcal{A},\mathcal{B}$ are combinations of open covers, dense open families and dense sets.

The selection principle theory was first considered in bitopological spaces by Kočinac and Özçağ in [15, 16] and they carried out a systematic study on selection principles mainly selective versions of separability in bitopological spaces, particularly in the space $C(X)$ of all continuous real-valued functions defined on a Tychonoff space X, where $C(X)$ is endowed with the topology τ_p of pointwise topology and the compact-open topology τ_k.

In the present paper we investigate some properties of bitopological selective versions of separability in function spaces and the set-open topologies will be the main tool.

The set-open topology on a family λ of nonempty subsets of a set X is a generalization of the compact-open topology (and of the topology of pointwise convergence). This notion was first introduced by Arens and Dugundji in [1] and was widely investigated by Osipov in [20–22]. In the next section we recall some facts on the set-open topologies.

For background material on selection principles we refer to the survey papers [13,27,30], for undefined notions in function spaces, see [2]. We will follow [8] for topological terminology and notations.

2. Main definitions and notations

In this paper, we consider the space $C(X)$ of all real-valued continuous functions defined on a Tychonoff space X.

Recall that a subset A of a space X is a *C-compact subset of* X if for any real-valued function f continuous on X, the set $f(A)$ is compact in $\mathbb{R}$.

A family λ of C-compact subsets of X is said to be hereditary with respect to C-compact subsets if it satisfies the following condition: whenever $A \in \lambda$ and B is a C-compact (in X) subset of A, then $B \in \lambda$. Recall that a family λ of nonempty subsets of a topological space (X,τ) is called a *π-network* for X if for any nonempty open set $U \in \tau$, there exists $A \in \lambda$ such that $A \subseteq U$.