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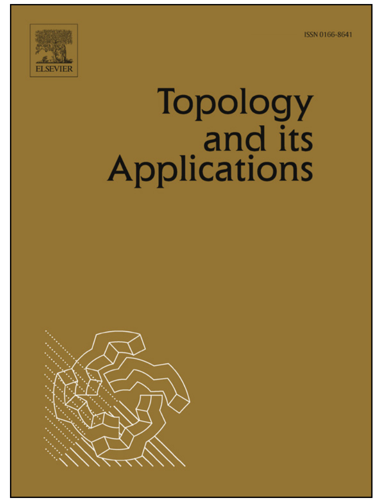

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H-CLOSED QUASITOPOLOGICAL GROUPS

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Abstract. An H-closed quasitopological group is a Hausdorff quasitopological group which is contained in each Hausdorff quasitopological group as a closed subspace. We obtained a sufficient condition for a quasitopological group to be H-closed, which allowed us to solve a problem by Arhangel'skii and Choban and to show that a topological group G is H-closed in the class of quasitopological groups if and only if G is Raĭkov-complete. Also we present examples of non-compact quasitopological groups whose topological spaces are H-closed.

One of the functions of a theory is to establish a correspondence between outer relations of an object with other objects and inner properties of the object. Now we proceed to concrete objects from general topology and topological algebra.

Further we shall follow the terminology of [5, 14]. In this paper term "space" means a Hausdorff topological space.

If Y is a subspace of a topological space X and $A \subseteq Y$, then by $\mathrm{cl}_Y(A)$ and $\mathrm{int}_Y(A)$ we denote closure and interior of A in Y, respectively. By $\mathbb{R}$, $\mathbb{Q}$ and $\mathbb{N}$ we denote the set of real, rational, and positive integer numbers, respectively.

It is well known that a compact space X is a closed subspace of any Hausdorff space which contains X. So we define a Hausdorff space X to be *H-closed* provided X is a closed subspace of any Hausdorff space which contains X. So H-closedness is an outer relation of a space. But it turned to be equivalent to an inner property of a space.

Theorem 1 ([14, Exercise 3.12.5], [2], (announcement in [1] and [3])). *For a Hausdorff space X the following conditions are equivalent:*

1. *the space X is H-closed;*
2. *for every family $\{V_s \colon s \in S\}$ of open subsets of X which has the finite intersection property the intersection $\bigcap\{\mathrm{cl}_X(V_s) \colon s \in S\}$ is non-empty;*
3. *every ultrafilter in the family of all open subsets of X converges;*
4. *every open cover $\{U_s \colon s \in S\}$ of the space X contains a finite subfamily $\{U_{s_1}, U_{s_2}, \ldots, U_{s_n}\}$ such that $\mathrm{cl}_X(U_{s_1}) \cup \mathrm{cl}_X(U_{s_2}) \cup \cdots \cup \mathrm{cl}_X(U_{s_n}) = X$.*

A regular space is H-closed if and only if it is compact, but there exists a non-regular H-closed space (see [1]).

A *semitopological group* consists of a group G and a topology τ on the set G such that the group operation $\cdot \colon G \times G \to G$ is separately continuous. A semitopological group with continuous inversion $\mathbf{inv} \colon G \to G \colon x \mapsto x^{-1}$ is called a *quasitopological group*. Also, a semitopological group (resp., a quasitopological group) with continuous group operation is called a *paratopological group* (resp., a *topological group*).

We shall say that a semitopological group G is *H-closed* in a class of Hausdorff semitopological groups $\mathscr{S}$ if G is a closed subgroup of every semitopological group $H \in \mathscr{S}$ which contains G as a subgroup. A topological group which is H-closed in the class of all topological group is called *absolutely closed*.

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