ARTICLE IN PRESS
APAL:2579

Annals of Pure and Applied Logic ••• (••••) •••–•••

Contents lists available at ScienceDirect

Annals of Pure and Applied Logic

www.elsevier.com/locate/apal

On generalized Van Benthem-type characterizations

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A R T I C L E I N F O

Article history:
Received 14 March 2016
Received in revised form 20 December 2016
Accepted 1 March 2017
Available online xxxx

MSC:
03C40
03C50
03B20
03B45

Keywords:
Model theory
Intensional logic
Propositional logic
Asimulation
Bisimulation
Van Benthem's theorem

A B S T R A C T

The paper continues the line of [6–8]. This results in a model-theoretic characterization of expressive powers of arbitrary finite sets of guarded connectives of degree not exceeding 1 and regular connectives of degree 2 over the language of bounded lattices.

This paper is a further step in the line of our enquiries into the expressive powers of intuitionistic logic and its extensions. This line started in late 2011, when we began to think about possible modifications of bisimulation relation in order to obtain the full analogue of Van Benthem modal characterization theorem for intuitionistic propositional logic. For the resulting modification, which was published in [6], we came up with a term "asimulation", since one of the differences between asimulations and bisimulations was that asimulations were not symmetrical.

Later we modified and extended asimulations in order to capture the expressive powers of first-order intuitionistic logic (in [7]) and some variants of basic modal intuitionistic logic (in [8]) viewed as fragments of classical first-order logic. Some other authors were also working in this direction; e.g. in [2] this line of research is extended to bi-intuitionistic propositional logic, although the author prefers directed bisimulations to asimulations.

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http://dx.doi.org/10.1016/j.apal.2017.03.002
0168-0072/

In the present paper, we generalize these results in the following way. We define the notion of a *guarded fragment* as a fragment of classical first-order logic (FOL) that can be naturally viewed as induced by some kind of intensional propositional logic via the corresponding notion of standard translation. Then we define a very general notion of asimulation which is applicable to an arbitrary guarded fragment of FOL. Finally, we isolate a rather wide subclass of guarded fragments, called *standard* guarded fragments, for which we show that they can express exactly first-order formulas which are invariant w.r.t. their corresponding version of asimulation notion.

Even though our general definition of asimulation notion is not, on the face of it, very similar to the definitions of different simulation notions present in the literature, we note that once a particular guarded fragment and a pair of models are given, one can rather straightforwardly localize the general definition of asimulation for this fragment transforming it into a well-known format based on a bunch of back and forth conditions very close to those known from the definition of bisimulations.[1] As the examples given in the end of Section 3 show, this localization process is actually rather conservative in that for some particular guarded fragments treated in the existing literature it gives out exactly the simulation notions that were already proved to characterize the expressive powers of these fragments. In this way, some earlier results on characterization of expressive power, including the original version of modal characterization theorem itself, are seen as but particular instances of a very general approach to defining simulation-like notions.

The group of propositional intensional logics that can be associated with some standard guarded fragment includes all of the above-mentioned logics (except, for obvious reasons, the first-order intuitionistic logic), but also many other formalisms.[2] It is worth noting that not all of these formalisms are actually extensions of intuitionistic logic, in fact, even the classical modal propositional logic which is the object of the original modal characterization theorem,[3] is also in this group. Thus the generalized asimulations defined in this paper have an equally good claim to be named generalized bisimulations, and if we still continue to call them asimulations, we do it mainly because for us these relations and their use are inseparable from the above-mentioned earlier results on the expressive power of intuitionistic logic.

The rest of this paper has the following layout. Section 1 fixes the main preliminaries in the way of notation and definition. In Section 2 we give some simple facts about Boolean functions and define the notion of a standard fragment. In Section 3 we do the main technical work preparing the 'easy' direction of our generalization of Van Benthem modal characterization theorem and define our central notion of (generalized) asimulation. In Section 4 we do the technical work for the 'hard' direction which mainly revolves around the properties of asimulations over ω-saturated models. Section 5 contains the proof of the result itself, and Section 6 gives conclusions and discusses the limitations of the result presented and prospects for future research.

For many crucial lemmas proven in Sections 3 and 4, the proofs are done by considering cases. Thus these proofs tend to be somewhat repetitive and lengthy, especially given that in some proofs the number of the cases to be considered is rather big. In order to keep the paper readable, we confined ourselves to treating a reasonably diverse partial selection of cases in the main text, whereas the other cases were transferred to Appendix A. Appendix B gives a glossary of some terms and notation pieces specific to the present paper, which may turn out to be helpful given the length of the main text.

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[1] The result of this transformation, however, will be uniform for any given pair of models M_1 and M_2, so that this localized definition can be considered as a general definition of asimulation corresponding to a given guarded fragment. In much the same manner, definitions of bisimulation often define bisimulations as relations *between a given pair of models*, which models act as parameters and do not affect the form of back-and-forth conditions themselves.

[2] For example, the fragment of intuitionistic propositional logic given by $\{\wedge, \vee, \bot, \top, \sim\}$, where $\sim$ is the intuitionistic negation, also corresponds to a standard fragment of FOL.

[3] For its formulation see, e.g. [3, Ch.1, Th. 13].