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Homology groups of types in stable theories and the Hurewicz correspondence ☆

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We give an explicit description of the homology group $H_n(p)$ of a strong type p in any stable theory under the assumption that for every non-forking extension q of p the groups $H_i(q)$ are trivial for $2 \le i < n$. The group $H_n(p)$ turns out to be isomorphic to the automorphism group of a certain part of the algebraic closure of n independent realizations of p; it follows from the authors' earlier work that such a group must be abelian. We call this the "Hurewicz correspondence" by analogy with the Hurewicz Theorem in algebraic topology.

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The present paper is a part of the project to study type amalgamation properties in first-order theories by means of homology groups of types. Roughly speaking (more precise definitions are recalled below in Section 1), a strong type p is said to have *n-amalgamation* if commuting systems of elementary embeddings among algebraic closures of proper subsets of the set of n independent realizations of p can always be extended to the algebraic closure of all n realizations. The type p has *n-uniqueness* if this extension is essentially unique. Generalized amalgamation properties for systems of *models* were introduced by Shelah in [11] and played an important role in [12]. The type amalgamation properties were studied extensively

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by Hrushovski in [9] and applications were given. In fact, the type amalgamation properties have been used in model theory at least as far back as Hrushovski's classification of trivial totally categorical theories in [8].

In the previous paper [4], we introduced a notion of homology groups for a complete strong type in any stable, or even rosy, first-order theory. The idea was that these homology groups should measure information about the amalgamation properties of the type p. We proved that if p has n-amalgamation for all n, then $H_n(p) = 0$ for every n, and that the failure of 4-amalgamation (equivalently, the failure of 3-uniqueness) over $\mathrm{dom}(p)$ in a stable theory corresponds to non-triviality of the group $H_2(p)$. Furthermore, we established in that paper that $H_2(p)$ is isomorphic in stable theories to a certain automorphism group of closures of realizations of p. In [5], the theory is developed in a more general context of *amenable collection of functors*.

The current paper generalizes the main results of [4] in the stable context: if the type p does not have $(n+2)$-amalgamation, then for some i with $2 \leq i \leq n$ and some nonforking extension p' of p, the group $H_i(p')$ must be nonzero. Furthermore, at the first i for which such p' with $H_i(p') \neq 0$ exists, we show that $H_i(p')$ is isomorphic to a certain automorphism group $\Gamma_i(p')$, which immediately implies that $H_i(p')$ is a profinite group.

To structure the proof in a more transparent way, we state a technical lemma (Lemma 2.3) in Section 2 of this paper and prove the main result (Theorem 2.1) of the paper using the lemma. The proof of Lemma 2.3 uses certain algebraic structures (n-polygroupoids) that were linked to failure of $(n+2)$-amalgamation in the previous paper [6]; but neither the statement of the lemma, nor the proof of Theorem 2.1 from the lemma use these structures.

The proof of Lemma 2.3 is contained in Section 3 of the paper. It turns out that most of the results of [6] do not need global n-amalgamation assumptions, only the amalgamation properties for the type p and its non-forking extensions. The additional work to verify the results of [6] is contained in Section 4.

1. Notation and preliminaries

In this paper, we always work with a fixed complete stable theory $T = T^{\mathrm{eq}}$ in a language $\mathcal{L}$, its saturated model $\mathcal{M} = \mathcal{M}^{\mathrm{eq}}$, and a complete type p over a small set $B = \mathrm{acl}(B)$. Throughout this paper, "independence" means nonforking independence. We use the usual notational conventions of stability theory, plus some further conventions from [4] and [6], some of which we will now recall.

Tuples of elements of the monster model or of variables will be denoted by lower-case letters (without a bar); the upper-case letters will denote sets. The base set B of the type p we have fixed is included in all algebraic closures. Algebraic closures will be denoted by a bar; for example, given a tuple c, the symbol $\bar{c}$ denotes $\mathrm{acl}(cB)$.

For sets A and C, the symbol $\mathrm{Aut}(A/C)$ denotes the group of elementary maps over C (i.e., fixing C pointwise) from $A \cup C$ onto $A \cup C$. For a type $q = \mathrm{tp}(a/C)$ with solution set A, $\mathrm{Aut}(q) := \mathrm{Aut}(A/C)$.

For the fixed type p, the symbol $p^{(k)}$ will denote the complete type of k independent realizations of the type p.

1.1. *Amalgamation properties and the homology groups*

We start by recalling from [3] and [4] the definition of a closed independent functor, which we then use to define both the type amalgamation properties and the homology groups of the type p.

A family of sets X ordered by inclusion can be endowed with a natural poset category structure: the objects are the elements of X and the morphisms are single inclusion maps $\iota_{u,v} : u \to v$ between any two sets $u, v \in X$ with $u \subseteq v$. The set X is called *downward-closed* if whenever $u \subseteq v \in X$, then $u \in X$.

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