ARTICLE IN PRESS APAL:2540

Annals of Pure and Applied Logic ••• (••••) •••–•••

Contents lists available at ScienceDirect

Annals of Pure and Applied Logic

www.elsevier.com/locate/apal

Reasoning about equilibria in game-like concurrent systems

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A R T I C L E I N F O

Article history:
Available online xxxx

MSC:
03B44
03B70
68T27
91A40

Keywords:
Games and strategic reasoning
Temporal logic
Equilibria

A B S T R A C T

In this paper we study techniques for reasoning about game-like concurrent systems, where the components of the system act rationally and strategically in pursuit of logically-specified goals. Specifically, we start by presenting a computational model for such concurrent systems, and investigate its computational, mathematical, and game-theoretic properties. We then define and investigate a branching-time temporal logic for reasoning about the equilibrium properties of game-like concurrent systems. The key operator in this temporal logic is a novel path quantifier $[\mathbf{NE}]\varphi$, which asserts that φ holds on all Nash equilibrium computations of the system.

1. Introduction

Our goal in this paper is to develop a theory and techniques for reasoning about *game-like concurrent systems*: concurrent systems in which system components (agents) act strategically in pursuit of their own interests. Game theory is the mathematical theory of strategic interaction, and as such is an obvious candidate to provide the analytical tools for this purpose [43]. However, since the systems we are interested in modelling and reasoning about are interacting computer programs, it seems appropriate to consider how existing techniques for the analysis of computer systems might be combined with game-theoretic concepts. Temporal logics [13] and model checking [10] form the most important class of techniques for reasoning about computer programs, and in this paper we are concerned with extending such formalisms and techniques to the game-theoretic analysis of systems.

The artificial intelligence, computer science, and multi-agent systems literatures contain a great deal of work on logics intended for reasoning about game-like systems: e.g., Parikh's Game Logic was an early example [44], and more recently ATL [3] and Strategy Logic [7] have received much attention. However, these formalisms are primarily intended for reasoning about the strategies/choices of players and their effects, rather than the preferences of players and the strategic choices they will make arising from them. It is, of

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http://dx.doi.org/10.1016/j.apal.2016.10.009

course, possible to use ATL or Strategy Logic (or indeed LTL, CTL, ...) to define the goals of agents and their preferences; but such languages do not provide any object language constructs for reasoning about the behaviour of such agents under the assumption that they act rationally and strategically in pursuit of their goals. In this paper, we present a branching time logic that is explicitly intended for this purpose. Specifically, we provide a logic for reasoning about the *equilibrium* properties of game-like concurrent systems.

Equilibrium concepts are the best-known and most widely applied analytical tools in the game theory literature, and of these Nash equilibrium is the best-known [43]. A Nash equilibrium is an outcome that can be obtained when no player has an incentive to deviate, i.e., to change its strategy. If we consider Nash equilibrium in the context of game-like concurrent systems, then it is natural to ask *which computations (runs, histories, ...) will be generated in equilibrium*? In [20], this question was investigated using the Iterated Boolean Games (iBG) model. In this model, each player is assumed to control a set of Boolean variables, and the game is played over an infinite sequence of rounds, where at each round every player chooses values for its variables. Each player has a goal, expressed as an LTL formula, and acts strategically in pursuit of this goal. Given this, some computations of a game can be identified as being the result of Nash equilibrium strategies, and [20] suggested that the key questions in the strategic analysis of the system are whether a given LTL formula holds in some or all equilibrium computations.

While the iBG model of [20] is useful for the purposes of exposition, it is not a realistic model of concurrent programs. Moreover, [20] provides no language for reasoning *about* the equilibria of systems: such reasoning must be carried out at the meta-level. This paper fills those gaps. First, we present a computational model that is more appropriate for modelling concurrent systems than the iBG model. In this model, the goals (and thus preferences) of players are given as temporal logic formulae that the respective player aspires to satisfy. After exploring some properties of this model, we introduce *Equilibrium Logic* (EL) as a formalism for reasoning about the equilibria of such systems. EL is a branching time logic that provides a new path quantifier $[\mathbf{NE}]\varphi$, which asserts that φ holds *on all Nash equilibrium computations of the system*. Thus, EL supports reasoning about equilibria directly in the object language. We then investigate some properties of this logic.

In particular in this paper we show that via a logical characterisation of equilibria in infinite games we can check useful properties of strategy profiles. We consider four logics for players' goals: LTL [45], CTL [9], the linear-time μ-calculus [51], and the modal μ-calculus [28]. Based on our logical characterisation, three problems are studied: Strategy-Checking, NE-Checking, and Equivalence-Checking, all of which are shown to be in PSPACE or in EXPTIME depending on the particular problem and temporal logic at hand. We also study the computational complexity of checking equilibrium properties, which can be expressed in the object language of EL. We show that the problem is 2EXPTIME-hard, even for LTL or CTL goals. This result shows, in turn, that checking equilibrium properties is equally hard in the linear-time and in the branching-time frameworks. We then investigate the complexity of model checking equilibrium computations with respect to dominant strategy equilibrium, a much stronger solution concept than Nash equilibrium. A summary of these complexity results is given at the end. We also present a class of games—where players are allowed to have an ordered set of (temporal logic) goals they want to see satisfied—for which all the main complexity results in the paper can be extended.

Structure of the paper. Section 2 defines the structure we use to represent games and strategies. Section 3 defines the model of games and strategies we will use in this paper. Section 4 gives a logical characterisation of equilibrium computations and investigates its main computational questions. Section 5 introduces our Equilibrium logics, and Section 6 presents a number of examples. Then, Section 7 studies the complexity of evaluating the new modal equilibrium operator, and Section 8 extends the main complexity results from Nash to dominant strategy equilibrium and from the standard model of games we use throughout the paper to a more general model where players are allowed to have an ordered set of temporal logic goals. At the end, Section 9 provides conclusions and related work, and Section 10 outlines a number of different avenues for further developments as well as some ideas underlying ongoing work. Throughout the paper, we assume

Please cite this article in press as: J. Gutierrez et al., Reasoning about equilibria in game-like concurrent systems, Ann. Pure Appl. Logic (2016), http://dx.doi.org/10.1016/j.apal.2016.10.009