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Hierarchical beam search for solving most relevant explanation in Bayesian networks

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A B S T R A C T

Most Relevant Explanation (MRE) is an inference problem in Bayesian networks that finds the most relevant partial instantiation of target variables as an explanation for given evidence. It has been shown in recent literature that it addresses the overspecification problem of existing methods, such as MPE and MAP. In this paper, we propose a novel hierarchical beam search algorithm for solving MRE. The main idea is to use a second-level beam to limit the number of successors generated by the same parent so as to limit the similarity between the solutions in the first-level beam and result in a more *diversified* population. Three pruning criteria are also introduced to achieve further diversity. Empirical results show that the new algorithm outperforms local search and regular beam search.

1. Introduction

Bayesian networks represent the conditional independences between random variables as directed acyclic graphs and provide standard approaches for solving inference problems, such as Most Probable Explanation (MPE), Maximum a Posterior (MAP), and Most Relevant Explanation (MRE). MPE [12] is the problem of finding the most likely instantiation of a set of target variables given the remaining variables as evidence. While in an MAP [12] problem, there are auxiliary variables besides target and evidence variables. Both MPE and MAP are solved by optimizing the joint posterior probability of the target variables given the evidence. As methods for explaining evidence [9,8], however, MPE and MAP often produce overspecified explanations [11,15], i.e., irrelevant target variables may also be included. This is because the criterion of maximizing a joint probability cannot exclude irrelevant target variables directly.

MRE [14,15] is an inference method developed to address the limitations of MPE and MAP. The key idea of MRE is to find a partial instantiation of the target variables that maximizes the Generalized Bayes Factor (GBF) [4,6] as an explanation for the evidence. GBF is a rational function of probabilities that is

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suitable for comparing explanations with different cardinalities. Theoretically, MRE is shown able to prune away independent and less relevant variables from the final explanation. The theoretical results have also been confirmed by a recent human study [11]. Because of the enlarged search space, however, MRE is shown to be extremely difficult to solve [16]. Exact algorithms developed in [18,20] can only scale to relatively small Bayesian networks. More difficult problems have to be solved by approximate MRE algorithms based on local search and Markov chain Monte Carlo methods [15]. Although efficient, these algorithms tend to find only local optimum solutions.

In this paper, we propose a novel *hierarchical beam search* algorithm to improve the accuracy of existing approximate MRE algorithms. The key idea is to use two levels of beams to increase the *diversity* of the solution population under consideration. The first-level beam is used to limit the search to the most promising solutions, similar to the regular beam search [13]. The second-level beams are newly introduced to limit the number of successors generated by a current solution to prevent over-reproduction of similar offsprings. Three pruning criteria based on the theoretical properties of MRE are also introduced to achieve further diversity as well as efficiency. We applied the new algorithm to solve MRE problems in a set of benchmark diagnostic Bayesian networks. Empirical results show that the new algorithm typically outperforms local search and regular beam search. In the experiments, we further evaluated the potential power of hierarchical beam search based on ensemble strategy. We also pointed out promising future research directions.

The rest of the paper is organized as follows. The background of the MRE problem and the beam search algorithm are introduced in Section 2. In Section 3, we discuss the proposed hierarchical beam search algorithm. The experimental results are evaluated in Section 4. Further discussions and future directions are provided in Section 5. Finally, conclusions are drawn in Section 6.

2. Background

A Bayesian network [12,7] is represented as a Directed Acyclic Graph (DAG). The nodes in the DAG represent random variables. The lack of arcs in the DAG defines conditional independence relations among the nodes. If there is an arc from node Y to X, i.e., $Y \to X$, we say that Y is a parent of X, and X is a child of Y. (We use upper-case letters to denote variables X or variable sets $\mathbf{X}$, and lower-case letters for values of scalars x or vectors $\mathbf{x}$.) A variable X is conditionally independent of its non-descendants given its parent set PA_X, which can be quantified by the conditional distribution $p(X|PA_X)$. The Bayesian network as a whole represents the joint probability distribution of $\prod_X p(X|PA_X)$.

MRE [15] has been developed to find a partial instantiation of the target variables as an explanation for given evidence in a Bayesian network. Here, *explanation* refers to the explanation of evidence, whose goal is to explain why some observed variables are in their particular states using the target variables in the domain. The complexity of MRE is conjectured to be NP^{PP} complete [16]. Formally, the MRE problem is defined as follows.

Definition 1. Let $\mathbf{M}$ be a set of targets, and $\mathbf{e}$ be the given evidence in a Bayesian network. Most Relevant Explanation is the problem of finding a partial instantiation $\mathbf{x}$ of $\mathbf{M}$ that has the maximum generalized Bayes factor score $GBF(\mathbf{x};\mathbf{e})$ as the explanation for $\mathbf{e}$, i.e.,

$$MRE(\mathbf{M};\mathbf{e}) = \arg\max_{\mathbf{x},\emptyset\subset\mathbf{X}\subseteq\mathbf{M}} GBF(\mathbf{x};\mathbf{e}), \tag{1}$$

where GBF is defined as

$$GBF(\mathbf{x};\mathbf{e}) = \frac{p(\mathbf{e}|\mathbf{x})}{p(\mathbf{e}|\bar{\mathbf{x}})}. \tag{2}$$

In the above equations, $\mathbf{x}$ is an instantiation of $\mathbf{X}$. We use $\bar{\mathbf{x}}$ to represent all of the alternative explanations of $\mathbf{x}$. To further study the properties of generalized Bayes factor, we can reformulate GBF as follows.