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On harmony and permuting conversions

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ABSTRACT

The paper exposes the relevance of permuting conversions (in natural-deduction systems) to the role of such systems in the theory of meaning known as proof-theoretic semantics, by relating permuting conversion to harmony, hitherto related to normalisation only. This is achieved by showing the connection of permuting conversion to the general notion of canonicity, once applied to arbitrary derivations from open assumption. In the course of exposing the relationship of permuting conversions to harmony, a general definition of the former is proposed, generalising the specific cases of disjunction and existential quantifiers considered in the literature.

1. Introduction

In the process of normalising (either weakly or strongly) derivations in intuitionistic propositional logic [14], based on Gentzen's natural-deduction proof-system NJ, two[1] kinds of transformations of derivations are employed.

Reduction: The removal of *maximal formulas* (detour elimination)
Permuting conversions: The reduction of the length of *maximal segments*

In this paper, I have two, rather modest, purposes.

1. While it has been well recognised within the theory of meaning known as *proof-theoretic semantics* (PTS) (see [19] for a summary and [5] for a detailed exposition, and Appendix A for a very brief delineation) that reductions are intimately associated with canonicity of derivations, and thereby with meaning, it seems that no *explicit* connection of permuting conversions to meaning has been pointed out. See Section 3 for a detailed analysis of one case where one would expect such a connection to be made, but

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[1] I ignore here *simplification conversions*, that are orthogonal to my current concern.

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finds it absent. I connect explicitly permuting conversion to meaning determination (via canonicity of derivations) and explain why this connection went unnoticed.

2. I found in the literature no *structural* definition of permuting conversion *in general*, not relating them to any specific connectives or quantifiers. The only available ones are related to disjunction and existential quantifier. One exception arises with general elimination rules, but again confined to the specific operators of intuitionistic logic (for example, see [12], pp. 190–194, or [20]). Even there, the specific permuting conversions presented are related to the relationship between normal ND-derivations and cut-free sequent-calculus derivations. There is no connection of those permuting conversions to the meaning determination by meaning-conferring rules of those intuitionistic connectives. This omission is in contrast to the role general-elimination rules fill in PTS in relation to harmony. See, for example, [16,17,8] among many others.

   I provide such a *general structural definition* of permuting conversions, which is connected with meaning determination.

2. Preliminaries

Ever since Prior's attack on the PTS programme[2] [15] it became evident that not every natural-deduction[3] (ND) proof-system can qualify as *meaning-conferring*. A major requirement from a meaning-conferring ND-system, going under the name of *harmony* and *stability* [3], requires a *balance* between the introduction rules (I-rules) and elimination rules (E-rules): neither group should overpower the other.

The harmony "half", namely that the I-rules are not too strong in comparison with the E-rules, was expressed by Prawitz [14] in his *inversion principle*:

> "Let α be an application of an elimination rule that has B as consequence. Then, deductions that satisfy the sufficient condition [...] for deriving the major premiss of α, when combined with deductions of the minor premisses of α (if any), already 'contain' a deduction of B; the deduction of B is thus obtainable directly from the given deductions without the addition of α."

If this principle is adhered to by an ND-system, this means that nothing can be "gained" by introducing some formula and then eliminating it: the conclusion is already "contained" in the derivations of the premises of the I-rules and can be obtained without this superfluous introduction followed by elimination. I take it that the term 'sufficient conditions' as used in the formulation of the inversion principle means premises of an I-rule. More on the inversion principle and its history can be found in [11].

The question now is, how can this "containment" be established?

Let $\mathcal{N}$ be an ND-system for some not further specified logic, intended to be meaning-conferring for that logic. Assume that the underlying object language has a well-defined notion of a *main* (or *principal*) operator of a formula. The common view is that the above mentioned "containment" is established by means of transformations on the $\mathcal{N}$-derivations known as *reductions*, eliminating occurrences of maximal formulas, the same transformation that the iteration of which leads to normalisation.

Definition 1 *(Maximal formula).* An occurrence of a formula φ in an $\mathcal{N}$-derivation $\mathcal{D}$ is *maximal* iff it is the conclusion of an application of an I-rule (of its principal operator), and the major premise of an application of an E-rule (for the same operator).

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[2] Prior did not refer, of course, to PTS, a much later term. He attacked the idea of defining meaning just by rules.

[3] See [5] for a detailed exposition of ND-systems and their use as meaning-conferring, definitional tool.