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On expressive power of basic modal intuitionistic logic as a fragment of classical FOL

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Abstract. The modal characterization theorem by J. van Benthem characterizes classical modal logic as the bisimulation invariant fragment of first-order logic. In this paper, we prove a similar characterization theorem for intuitionistic modal logic. For this purpose we introduce the notion of modal asimulation as an analogue of bisimulations. The paper treats four different fragments of first-order logic induced by their respective versions of Kripke-style semantics for modal intuitionistic logic. It is shown further that this characterization can be easily carried over to arbitrary first-order definable subclasses of classical first-order models.

Keywords. model theory, modal logic, intuitionistic logic, propositional logic, bisimulation, Van Benthem's theorem.

1 Introduction

It is well known that Kripke semantics uses a rather limited set of first-order tools (including, first and foremost, restricted quantification over worlds) to interpret the language of basic modal logic. This allows to embed basic modal logic into the first-order logic (FOL) via the so called *standard translation* encoding a given modal propositional formula by a first-order one with a single free variable (this variable represents a world in which this modal formula is supposed to be true). In this way, basic modal logic is shown to correspond to a specific (so called *modal*) fragment of FOL which, in turn, is often called in this context the *correspondence language* for modal logic. Even though it is relatively easy to find out if a given first-order formula is a standard translation of a modal formula, it is not so easy to say when a first-order formula can be *defined* by a translation of a modal formula, which would amount to a description of expressive power of the modal fragment of FOL. A well-known answer to this more difficult question is given by the modal characterization theorem[1] proved by J. van Benthem. The answer is that a first-order formula with a single free variable is equivalent to a

[1]See, e.g. [3, Ch.1, Th. 13]. Another standard reference is [4], where the modal characterization theorem is proved as Theorem 2.68 in a very detailed and accessible way.