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A geometric principle of indifference

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ABSTRACT

That one's degrees of belief at any one time obey the axioms of probability theory is widely regarded as a necessary condition for static rationality. Many theorists hold that it is also a sufficient condition, but according to critics this yields too subjective an account of static rationality. However, there are currently no good proposals as to how to obtain a tenable stronger probabilistic theory of static rationality. In particular, the idea that one might achieve the desired strengthening by adding some symmetry principle to the probability axioms has appeared hard to maintain. Starting from an idea of Carnap and drawing on relatively recent work in cognitive science, this paper argues that conceptual spaces provide the tools to devise an objective probabilistic account of static rationality. Specifically, we propose a principle that derives prior degrees of belief from the geometrical structure of concepts.

1. The principle of indifference and its discontents

It is widely accepted that obedience to the axioms of probability theory (or probabilistic coherence) is a necessary condition for static rationality: one is rational at any given time only if at that time one's degrees of belief are representable by a probability function.[1] The best-known argument for this claim proceeds by showing that violation of the condition makes one vulnerable to so-called Dutch books, that is, collections of bets that appear fair as judged by one's degrees of belief but that jointly ensure a financial loss (cf. [48,14]). Of a more recent date is Joyce's [36] inaccuracy-minimization argument, which purports to show that for

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[1] Static rationality is commonly contrasted with dynamic rationality, which concerns the question of how a person's degrees of belief are to change in response to the receipt of new information. Most philosophers who subscribe to the view of static rationality described here hold that one is dynamically rational precisely if one changes one's degrees of belief via Bayes' rule, but a commitment to the former view certainly does not commit one to the latter; see [15,17,19]. The present paper will be exclusively concerned with static rationality.

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any degrees-of-belief function not satisfying the probability axioms there is a degrees-of-belief function that, by one's own lights, counts as more accurate (where accuracy is understood in terms of a so-called scoring rule).

If this is accepted, then an important next question is whether obeying probability theory is also *sufficient* for static rationality. A typical first inclination is to deny this. Surely there can be degrees-of-belief functions that are formally probability functions but that appear irrational from a pretheoretical viewpoint. For example, it is not necessarily probabilistically incoherent to believe to a low degree that Barack Obama is the current president of the United States, or to believe to a high degree that Iceland will win more gold medals than Great Britain in the next Olympics. Yet anyone who had roughly the same relevant evidence that we have and still held these degrees of belief would strike us as being—to say the least—not fully rational.

Motivated by examples of this kind, a number of attempts have been undertaken to strengthen the account of static rationality by adding further principles to the probability axioms. It is not our aim here to give an inventory of the various proposals that have been made in this connection. Rather, we want to focus on one principle that has come up time and again (under different names) in the course of the history of thinking about probabilistic coherence in relation to rationality, and that has an undeniable intuitive appeal. The principle concerns the determination of prior degrees of belief on grounds of symmetry. This principle, dubbed "the Principle of Insufficient Reason" by Laplace [42] and renamed as "the Principle of Indifference" by Keynes [40] (henceforth "POI"), says that given a set of mutually exclusive (at most one can be true) and jointly exhaustive (at least one must be true) propositions, and barring countervailing considerations, one ought to invest the same confidence in each of the propositions. Put differently, given a set of propositions of the aforementioned kind, if you lack any reason *not* to treat them evenhandedly, you *should* treat them evenhandedly.

At least at first sight, it seems that few things could be more reasonable than this principle. Consider a coin with an unknown bias for heads; all we know is that its bias for heads is a multiple of $^1/_{10}$. We thus have a set $\{H_i\}_{0\leqslant i\leqslant 10}$ of eleven mutually exclusive and jointly exhaustive bias hypotheses, with H_i the hypothesis that the coin has a bias for heads of $^i/_{10}$. The POI now implores us to believe each of these hypotheses to a degree of $^1/_{11}$. That is fully in accordance with our intuitions about the symmetry of the situation; barring evidence about the bias of the coin, any *non*-uniform assignment of degrees of belief would appear arbitrary in a way in which the uniform distribution appears not to be.

Though seemingly reasonable, Hawthorne et al. [32] are certainly right that the POI "is still viewed largely with suspicion." Indeed, the principle faces a problem that many consider to be fatal for it. The following illustrates the problem in its simplest form. Suppose you are about to draw a ball from an urn with many balls. At this point, you know nothing about the colors of the balls in the urn. To which degree should you believe that the ball you draw will be red? Here is a plausible answer, backed by the POI: you should believe that the ball will be red to a degree of $^1/_2$. After all, the ball will either be red or it will not be red. You have no reason to believe either of these possibilities to a greater degree than the other. So you should treat them evenhandedly. Unfortunately, the following train of thought appears equally legitimate: Either the ball will be red, or it will be blue, or it will be some other color. You have no reason *not* to treat these hypotheses evenhandedly, so you *should* treat them evenhandedly. Thus you should believe to a degree of $^1/_3$ that the ball will be red. One could go on in this way, arriving at still other answers, apparently all reasonable, to the question of what your degree of belief should be that the ball you are about to draw from the urn will be red.

That is not what we wanted. We wanted to arrive at the *one* reasonable answer that respected the symmetries of the situation. But the foregoing seems to show that what counts as the symmetries of a situation may depend on how we describe the situation, and that there can be more than one admissible way to describe a given situation, where the admissible ways may have different implications for what the symmetries of the situation are. Indeed, it has been shown in the literature that this problem is quite

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