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The structure of ideas in *The Port Royal Logic* ☆

John N. Martin

Department of Philosophy, University of Cincinnati, P.O. Box 210374, Cincinnati, OH 45221, United States

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ABSTRACT

This paper addresses the degree to which *The Port Royal Logic* anticipates Boolean Algebra. According to Marc Dominicy the best reconstruction is a Boolean Algebra of Carnapian properties, functions from possible worlds to extensions. Sylvain Auroux's reconstruction approximates a non-complemented bounded lattice. This paper argues that it is anachronistic to read lattice algebra into the *Port Royal Logic*. It is true that the *Logic* treats extensions like sets, orders ideas under a containment relation, and posits mental operations of abstraction and restriction. It also orders species in a version of the tree of Porphyry, and allows that genera may be divided into species by privative negation. There is, however, no maximal or minimal idea. Abstraction is not binary. Neither abstraction nor restriction is closed. Ideas under containment, therefore, do not form a lattice. Nor are the relevant formal properties of lattices discussed. Term negation is privative, not a complementation operation. The technical ideas relevant to the discussion are defined. The *Logic*'s purpose in describing structure was not to develop algebra in the modern sense but rather to provide a new basis for the semantics of mental language consistent with Cartesian metaphysics. The account was not algebraic, but metaphysical and psychological, based on the concept of *comprehension*, a Cartesian version of medieval objective being.

0. Introduction

This paper investigates the structure of ideas in *The Port Royal Logic*. It has two immediate goals. The first is to explain what this structure is and why. The second is theoretical, to assess the degree to which the concepts of structure employed approximate those of modern algebra. Doing so will amount to a

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E-mail address: john.martin@uc.edu.

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reexamination of the interpretations of the *Logic* by Sylvain Auroux and Marc Dominicy,[1] who argue that the structure of ideas in the *Logic* approximates that of Boolean algebra. In my opinion that claim is exaggerated. I hope to show that although there are novelties in the approach to structure, these are not algebraic. Rather, innovations derive not from mathematical developments, but rather from the need to rework various foundational concepts of medieval logic necessitated by Cartesian metaphysics and epistemology. To preserve the wider corpus of logical doctrines that had been accepted since the Middle Ages, the *Logic*'s authors found it necessary to redefine basic notions like signification. The rebuilding did have novel implications for the structure of ideas, but, as I hope to show, these were extensions of and changes in earlier doctrines. They were not couched in the language of mathematics but rather in that of medieval psychology and metaphysics.

In the discussion I will make use of concepts from modern logic, but today's logic can be applied to history is different ways. In rare cases a modern concept fits an earlier one exactly. An example is Bocheński's observation that Philo's 4th century B.C. implication is the same as the material conditional, which he then illustrated by a modern truth-table.[2] Let us call an interpretation that identifies an historical concept with a modern one a *paraphrase*. We shall see that Auroux and Dominicy paraphrase some of the *Logic*'s operations in terms of Boolean algebra.[3]

More commonly an earlier account is explained by extending it, reformulating both the original and the extension in modern terms. Bits and pieces of an earlier doctrine can be recast into modern vocabulary and elaborated in a way that results in a recognizable theory according to modern standards. Let us call such an interpretation an *extension*. Natural deduction completeness proofs of Aristotle's syllogistic are examples. Some of the points I will make below about the *Logic* will count as extensions in this sense.

A third type of reading uses modern logic to correct an earlier version. Typically these are extensions with revisions. The revision may clarify poorly defined terms, reorganize definitional order, or correct inconsistencies. Let us call a reading of this sort a *reconstruction*. Because Auroux and Dominic both find the treatment of term negation in the *Logic* to be flawed, the interpretations they give count as reconstructions in this sense.

The central topic of the paper is the degree to which the structure of ideas in the *Logic* approximates that of modern algebra. The key concepts from modern algebra to which this structure will be compared are *partial ordering, lattice* and *Boolean algebra*. Since the paper addresses historians as well as logicians, some introduction to the technical material is in order.

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[1] Auroux [7], Dominicy [14]. There have been other skeptics about how much the 17th century work anticipates modern logic. See, for example, Conimbricenses [12], pp. 318–320, and Pariente [21], p. 246, who writes:

> L'originalité du livre ne réside pas, il est vrai dans ses innovations formelles. Arnauld et Nicole ne sont pas des inventeurs sur le plan du calcul logique. Rien n'est plus éloigné de leur style de réflexion que les efforts diversifiés et inlassables d'un Leibniz pour mettre sur pied un formalisme efficace et rationnel.

More directly relevant to this paper is Russell Wahl's judgment, "It is a mistake, I believe, to read into the *Logic* a prelude to set theory". Wahl [23], p. 673.

[2] Bocheński [8]. §§ 20.07 and 20.071, p. 117.

[3] We shall be making use of standard concepts from set theory and abstract algebra. Those from algebra are defined as follows. $\langle B, \leq \rangle$ is a *partial ordering* iff $\leq$ is a reflexive, transitive and anti-symmetric binary relation on B. If $\langle B, \leq \rangle$ and $\langle B', \leq' \rangle$ are partial orderings, a function f from B into B', f is said to be *monotonic* iff, for any $x, y \in B$, if $x \leq y$ then $f(x) \leq' f(y)$; *antitonic* iff, for any $x, y \in B$, if $x \leq y$ then $f(x) \geq' f(y)$; and B is *dual to* B' *relative to* f iff f is onto and antitonic. $\langle B, \wedge \rangle / \langle B, \vee \rangle$ is a *meet/join semi-lattice* iff B is closed under a binary operation $\wedge/\vee$ that is associative, commutative, and idempotent. An ordering relation $\leq$ on a *meet/join* semi-lattice B is defined as follows: $x \leq y$ iff $x \wedge y = x/x \vee y = y$. It follows that if $\langle B, \wedge \rangle / \langle B, \vee \rangle$ is a *meet/join semi-lattice*, then $\langle B, \leq \rangle$ is a partial ordering. $\langle B, \wedge, \vee \rangle$ is a *lattice* iff $\langle B, \wedge \rangle$ and $\langle B, \vee \rangle$ are *meet* and *join semi-lattices*. In a partial ordering $\langle B, \leq \rangle$ *the greatest lower bound* of $\{xy\}$, briefly $glb\{x, y\}$ if it exists, is the $z \in B$ such that $z \leq x$, $z \leq y$, and for any w in B, if $w \leq x$ and $w \leq y$, then $w \leq z$; *the least upper bound* of $\{x, y\}$, briefly $lub\{x, y\}$, if it exists, is the $z \in B$ such that $x \leq z$, $y \leq z$, and for any w in B, if $x \leq w$ and $y \leq w$, then $z \leq w$. It follows that $\langle B, \wedge, \vee \rangle$ is a lattice iff $\langle B, \leq \rangle$ is a partial ordering closed under $lub = \wedge$ and $glb = \vee$. 0 is *the least element* of a lattice B iff, $0 \in B$ and for any x in B, $0 \leq x$, $0 \wedge x = 0$ and $0 \vee x = x$; 1 is *the greatest element* iff $1 \in B$ and for any x in B, $x \leq 1$, $1 \wedge x = x$ and $1 \vee x = 1$.