

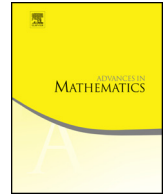


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Euler class groups and the homology of elementary and special linear groups



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ABSTRACT

We improve homology stability ranges for elementary and special linear groups over rings with many units. Our result implies stability for unstable Quillen K -groups and proves a conjecture of Bass. For commutative local rings with infinite residue fields, we show that the obstruction to further stability is given by Milnor–Witt K -theory. As an application we construct Euler classes of projective modules with values in the cohomology of the Milnor–Witt K -theory sheaf. For d -dimensional commutative noetherian rings with infinite residue fields we show that the vanishing of the Euler class is necessary and sufficient for an oriented projective module P of rank d to split off a rank 1 free direct summand. Along the way we obtain a new presentation of Milnor–Witt K -theory and of symplectic K_2 simplifying the classical Matsumoto–Moore presentation.

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1. Introduction

The purpose of this paper is to improve stability ranges in homology and algebraic K -theory of elementary and special linear groups, and to apply these results to construct obstruction classes for projective modules to split off a free direct summand.

Our first result concerns a conjecture of Bass [5, Conjecture XVI on p. 43]. In [5] he conjectured that for a commutative noetherian ring A whose maximal ideal spectrum has dimension d the canonical maps

$$\pi_i BGL_{n-1}^+(A) \rightarrow \pi_i BGL_n^+(A)$$

are surjective for $n \geq d + i + 1$ and bijective for $n \geq d + i + 2$. Here, for a connected space X , we denote by X^+ Quillen's plus-construction with respect to the maximal perfect subgroup of $\pi_1 X$, and we write $BGL_n^+(A)$ for $BGL_n(A)^+$. In this generality, there are counterexamples to Bass' conjecture; see [35, §8]. The best general positive results to date concerning the conjecture are due to van der Kallen [37] and Suslin [34]. They prove that the maps are surjective for $n - 1 \geq \max(2i, \text{sr}(A) + i - 1)$ and bijective for $n - 1 \geq \max(2i, \text{sr}(A) + i)$ where $\text{sr}(A)$ denotes the stable rank of A [39]. Here A need not be commutative nor noetherian.

In this paper we prove Bass' conjecture for rings with many units. Recall [28] that a ring A (always associative with unit) has many units if for every integer $n \geq 1$ there is a family of n *central elements* of A such that the sum of each non-empty subfamily is a unit. Examples of rings with many units are infinite fields, commutative local rings with infinite residue field and algebras over a ring with many units. Here is our first main result.

Theorem 1.1 (*Theorem 3.10*). *Let A be a ring with many units. Then the natural homomorphism*

$$\pi_i BGL_{n-1}^+(A) \rightarrow \pi_i BGL_n^+(A)$$

is an isomorphism for $n \geq i + \text{sr}(A) + 1$ and surjective for $n \geq i + \text{sr}(A)$.

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