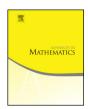


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## Factorization and non-factorization theorems for pseudocontinuable functions



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#### ABSTRACT

Let  $\theta$  be an inner function on the unit disk, and let  $K^p_\theta := H^p \cap \theta \overline{H^0_0}$  be the associated star-invariant subspace of the Hardy space  $H^p$ , with  $p \geq 1$ . While a nontrivial function  $f \in K^p_\theta$  is never divisible by  $\theta$ , it may have a factor h which is "not too different" from  $\theta$  in the sense that the ratio  $h/\theta$  (or just the anti-analytic part thereof) is smooth on the circle. In this case, f is shown to have additional integrability and/or smoothness properties, much in the spirit of the Hardy–Littlewood–Sobolev embedding theorem. The appropriate norm estimates are established, and their sharpness is discussed.

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#### 1. Introduction and results

The pseudocontinuable functions in the paper's title are the noncyclic vectors of the backward shift operator

$$S^*: f \mapsto \frac{f - f(0)}{z}$$

acting on the Hardy space  $H^2$  – or, more generally,  $H^p$  (see the definition below) – of the unit disk  $\mathbb{D} := \{z \in \mathbb{C} : |z| < 1\}$ . As usual, "noncyclic" means "lying in some proper (closed) invariant subspace", and a well-known result from [7] tells us that a function f is noncyclic for  $S^*$  if and only if it admits a pseudocontinuation to  $\mathbb{D}^- := \{z : |z| > 1\}$ . The latter means that there exists a meromorphic function of bounded characteristic in  $\mathbb{D}^-$  whose boundary values agree with f almost everywhere on the circle  $\mathbb{T} := \partial \mathbb{D}$ . The other key notion in this paper is smoothness, a phenomenon that can also be described in terms of a "pseudocontinuation", this time understood as pseudoanalytic extension in the sense of Dyn'kin (see [16]). The general principle, as explained in [16], is that a function f holomorphic on  $\mathbb{D}$  (and, say, continuous up to the boundary) will be smooth on  $\mathbb{T}$ , in some sense or other, if and only if it extends to  $\mathbb{D}^-$  as a  $C^1$  function whose Cauchy–Riemann  $\overline{\partial}$ -derivative becomes appropriately small – or does not grow too fast – near  $\mathbb{T}$ .

Thus, loosely speaking, we are concerned with the interplay of the two kinds of pseudo-continuation. In this connection, we also mention the survey paper [15] which summarizes some of the previous results pertaining to the two topics and discusses the interrelation-ship between them.

Now let us try and describe the setup more accurately. Recall, first of all, that the Hardy space  $H^p = H^p(\mathbb{D})$  with 0 is formed by those holomorphic functions <math>f on  $\mathbb{D}$  which satisfy

$$\sup_{0 < r < 1} \int_{\mathbb{T}} |f(r\zeta)|^p dm(\zeta) < \infty$$

(we write m for the normalized arclength measure on  $\mathbb{T}$ ), while  $H^{\infty} = H^{\infty}(\mathbb{D})$  stands for the space of bounded holomorphic functions. As is customary, we identify  $H^p$  functions with their boundary values (defined almost everywhere on the circle) and treat  $H^p$  as a subspace of  $L^p = L^p(\mathbb{T}, m)$ , endowed with the standard  $L^p$ -norm  $\|\cdot\|_p$ . Further, a function  $\theta \in H^{\infty}$  is said to be *inner* if  $|\theta| = 1$  almost everywhere on  $\mathbb{T}$ .

Beurling's famous theorem characterizes the invariant subspaces of the (forward) shift operator  $S: f \mapsto zf$  in  $H^2$  as those of the form  $\theta H^2$ , where  $\theta$  is an inner function; see, e.g., [17, Chapter II]. Accordingly, the  $S^*$ -invariant (or *star-invariant*) subspaces of  $H^2$  can be written as

$$H^2\ominus\theta H^2=:K^2_\theta,$$

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