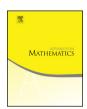


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Sharp bounds for the cubic Parsell–Vinogradov system in two dimensions [☆]



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ABSTRACT

We prove a sharp decoupling for a certain two dimensional surface in \mathbb{R}^9 . As an application, we obtain the full range of expected estimates for the cubic Parsell–Vinogradov system in two dimensions.

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1. Introduction and statements of main results

Fix $d, s \geq 1$ and $k \geq 2$. We use \mathbf{x} to denote the vector $(x_1, \ldots, x_d) \in \mathbb{R}^d$, and \mathbf{i} to denote the d-tuple (i_1, \ldots, i_d) of non-negative integers. The monomial $x_1^{i_1} \ldots x_d^{i_d}$ will be abbreviated to \mathbf{x}^i . Consider the integer solutions

$$\mathbf{x}_1, \mathbf{x}_2, \dots, \mathbf{x}_s, \mathbf{y}_1, \mathbf{y}_2, \dots, \mathbf{y}_s \tag{1.1}$$

of the system of Diophantine equations (often referred to as the *Parsell-Vinogradov* system)

$$\mathbf{x}_1^{\mathbf{i}} + \dots + \mathbf{x}_s^{\mathbf{i}} = \mathbf{y}_1^{\mathbf{i}} + \dots + \mathbf{y}_s^{\mathbf{i}}. \tag{1.2}$$

Here $0 \le i_1, i_2, ..., i_d \le k$ range through all possible integers such that $1 \le i_1 + i_2 + ... + i_d \le k$. For instance, when d = 1, the system (1.2) consists of the following k equations, known as the classical Vinogradov system

$$x_1^i + \dots + x_s^i = y_1^i + \dots + y_s^i$$
, with $1 \le i \le k$. (1.3)

When d = k = 2, the system (1.2) becomes

$$x_{1,1} + x_{1,2} + \dots + x_{1,s} = y_{1,1} + y_{1,2} + \dots + y_{1,s},$$

$$x_{2,1} + x_{2,2} + \dots + x_{2,s} = y_{2,1} + y_{2,2} + \dots + y_{2,s},$$

$$x_{1,1}^2 + x_{1,2}^2 + \dots + x_{1,s}^2 = y_{1,1}^2 + y_{1,2}^2 + \dots + y_{1,s}^2,$$

$$x_{2,1}^2 + x_{2,2}^2 + \dots + x_{2,s}^2 = y_{2,1}^2 + y_{2,2}^2 + \dots + y_{2,s}^2,$$

$$x_{1,1}x_{2,1} + x_{1,2}x_{2,2} + \dots + x_{1,s}x_{2,s} = y_{1,1}y_{2,1} + y_{1,2}y_{2,2} + \dots + y_{1,s}y_{2,s}.$$

$$(1.4)$$

For a large N, we let $J_{s,d,k}(N)$ denote the number of integer solutions (1.1) of the system of equations (1.2) satisfying $1 \le x_{1,j}, ..., x_{d,j}, y_{1,j}, ..., y_{d,j} \le N$ for each $1 \le j \le s$.

As far as we can tell, the investigation of the quantities $J_{s,d,k}(N)$ for $d \geq 2$ was initiated by Parsell in [12]. This paper also explains some of the motivation behind considering such quantities. For instance, one motivation comes from counting rational linear subspaces of a given dimension lying on the hyper-surface defined by

$$c_1 z_1^k + \dots + c_s z_s^k = 0, (1.5)$$

for given $c_1, \ldots, c_s \in \mathbb{Z}$. In order to apply the Hardy–Littlewood circle method, one needs a good upper bound for $J_{s,d,k}(N)$. We mention that a related Diophantine system was considered earlier in [1].

Parsell, Prendiville and Wooley [13] provided the following lower bound for $J_{s,d,k}(N)$.

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