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Residues for flags of holomorphic foliations

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ABSTRACT

In this work we prove a Baum–Bott type residue theorem for flags of holomorphic foliations. We prove some relations between the residues of the flag and the residues of their correspondent foliations. We define the Nash residue for flags and we give a partial answer to the Baum–Bott type rationality conjecture in this context.

Contents

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1. Introduction

A 2-flag of foliations is a pair of foliations $(\mathcal{F}_1, \mathcal{F}_2)$ such that the leaves of $\mathcal{F}_1$ are contained in the leaves of $\mathcal{F}_2$. We call $\mathcal{F}_1$ a subfoliation of $\mathcal{F}_2$. In this work, we considerer 2-flags formed by 2 holomorphic foliations on a complex manifold M of dimension n. Our main result is a Baum–Bott type residue theorem. We believe that the study of characteristic classes of singular flags of holomorphic foliations can be useful to give information about these structures. The study of characteristic classes of real foliations was firstly considered by Feigin [12], where he proposed two constructions for characteristic classes of flags in an attempt to answer a question about topological obstruction for integrability. Several other authors studied characteristic classes of flags, [8,11].

In the holomorphic context, flags of holomorphic foliations appear naturally on works about foliations with algebraic and rationally connected leaves: Miyaoka [18], Bogomolov–Mcquillan [2], and Kebekus–Sola Conde–Toma [16]. Other very important situation where appear flags of foliations is the so called Brunella's conjecture:

If $\mathcal{F}$ is a codimension one holomorphic foliation on $\mathbb{P}^3$, then $\mathcal{F}$ either admits an algebraic invariant surface or admits a subfoliation by algebraic curves.

In the case where $\mathcal{F}$ admits a subfoliation by algebraic curves we have a 2-flag. Then the study of the singularities of these foliations is important. Some authors studied flags of singular holomorphic foliations: in [10] Corrêa and Soares proved an inequality involving the degrees of two distributions (foliations) which form a flag on projective spaces. Mol in [19] studied the polar classes of flags of holomorphic foliations.

2. Baum–Bott residues theory for flags

The main results of this section are a Bott vanishing theorem and a Baum–Bott theorem for flags, where the first one is "finer" than the classical result of Bott. The reason is that our theorem detects some characteristic classes of flags that the Bott vanishing theorem for foliations does not detect. These classes are legitimate of flags, see Remark 1. Here we apply the localization theory of characteristic classes developed mainly by Lehmann and Suwa to localize the characteristic classes. For details on this theory we refer to [20].

2.1. Flags of holomorphic foliations

Let M be a connected complex manifold of dimension n. Let us denote by Θ_M the tangent sheaf of M.