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Bases for cluster algebras from orbifolds

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ABSTRACT

We generalize the construction of the bracelet and bangle bases defined in [36] and the band basis defined in [43] to cluster algebras arising from orbifolds. We prove that the bracelet bases are positive, and the bracelet basis for the affine cluster algebra of type $C_n^{(1)}$ is atomic. We also show that cluster monomial bases of all skew-symmetrizable cluster algebras of finite type are atomic.

Contents

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1. Introduction

Cluster algebras were introduced by Fomin and Zelevinsky [25] in the effort to understand a construction of canonical bases by Lusztig [34] and Kashiwara [30]. A cluster algebra is a commutative ring with a distinguished set of generators called *cluster variables*. Cluster variables are grouped into overlapping finite collections of the same cardinality called *clusters* connected by local transition rules which are determined by a skew-symmetrizable *exchange matrix* associated with each cluster, see Section 2 for precise definitions.

One of the central problems in cluster algebras theory is a construction of good bases. It was conjectured in [25] that these bases should contain cluster monomials, i.e. all products of cluster variables belonging to every single cluster. Linear independence of cluster monomials in skew-symmetric case was proved by Cerulli Irelli, Keller, Labardini-Fragoso and Plamondon in [7], for a general skew-symmetrizable case linear independence was recently proved by Gross, Hacking, Keel and Kontsevich in [28]. In the finite type cluster monomials themselves form a basis (Caldero and Keller [1]).

Bases containing cluster monomials were constructed for various types of cluster algebras. These include ones by Sherman and Zelevinsky [41] (rank two affine type), Cerulli Irelli and Esposito [4,6] (affine type $\widetilde{A}_2$), Ding, Xiao and Xu [12] (affine type), Dupont [13,15], Geiss, Leclerc and Schröer [27], Plamondon [40] (*generic* bases for acyclic types), Lee, Li and Zelevinsky (*greedy* bases in rank two algebras).

In [36] Musiker, Schiffler and Williams constructed two types of bases (*bangle* basis $\mathcal{B}^\circ$ and *bracelet* basis $\mathcal{B}$) for cluster algebras originating from unpunctured surfaces [23, 24,20]. A *band* basis (we call it $\mathcal{B}^\sigma$) was introduced by D. Thurston in [43]. All the three bases are parametrized by collections of mutually non-intersecting arcs and closed loops, and all their elements are *positive*, i.e. the expansion of any basis element in any cluster is a Laurent monomial with non-negative coefficients.

In the present paper, we extend the construction of all the three bases to cluster algebras originating from orbifolds.

Theorem 1.1. *Let $\mathcal{A}$ be a cluster algebra with principal coefficients constructed by an unpunctured orbifold with at least two boundary marked points. Then $\mathcal{B}$, $\mathcal{B}^\sigma$ and $\mathcal{B}^\circ$ are bases of $\mathcal{A}$.*

Our main tools are the tropical duality by Nakanishi and Zelevinsky [38], and the theory of unfoldings developed in [18,19]. The notion of an unfolding was introduced by Zelevinsky, it provides a reduction of problems on (certain) skew-symmetrizable cluster algebras to appropriate skew-symmetric ones. In our case, unfoldings allow us to treat